RĪGAS EKONOMIKAS AUGSTSKOLA
STOCKHOLM SCHOOL OF ECONOMICS IN RIGA

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Abstract

This paper tests the weak-form efficiency of the three Baltic Stock markets by using a moving average and the head-and-shoulders pattern. The rules are defined using fuzzy logic to better account for cognitive uncertainties associated with them. Significant risk-adjusted returns are found. A different contribution to theoretical research on technical analysis is clarification and improvements of certain aspects of the head-and-shoulders pattern recognition algorithm. Finally, certain trading results point to interesting irregularities specific to the Baltic Stock markets.

Introduction

On September 27th 2004 the Latvian and Estonian stock exchanges (RSE and TSE, respectively) first used the SAXESS trading system, which effectively integrated the Baltic stock market (BSM) into the Scandinavian trading system. Lithuania (VSE) joined ranks half a year later. The focal motive for integration was increased visibility in the global stock market, which would lead to higher market liquidity and depth. Parallel to integration, RSE and TSE initiated a liquidity provider program. These two initiatives increased volumes (as evident from the official statistics and press releases for 2004), and with it – volatility, as observable from noticeable fluctuations in the market.

However, these fluctuations were, and are, hard to interpret based on the conventional fundamental approach to valuing stocks. Take the LFO1L (Lifosa, VSE) – from February to November 2005, capitalization increased seven times, only to drop abruptly by 40 percent. The DPK1R (Ditton pievadķēžu rūpnīca, RSE) predicatively fell after each interim financial report, only to rebound to similar levels in the same month. Both examples are at odds with the notion of efficient markets. There are many similar examples of frenetic stock behavior in the context of the BSM, where fundamental analysis breaks down.

This may suggest (in parallel to fundamentals) behavioral factors at work. Technical analysis (TA), which focuses on the demand-supply relationship, and not intrinsic value, may thus be a profitable auxiliary trading strategy. We look into the unexplored field of applicability of chart patterns as a means of explaining price movements on the BSM.

Patterns are, however, in the eyes of the beholder. Even among each other, technical analysts tend to disagree on what the definitive characteristics of a pattern are; this makes academic analysis tricky. Recent adoption of control procedures within the field of financial analysis could help. In particular, fuzzy logic looks very promising. This allows taking into account of various interpretations of the same descriptive statement – e.g. Mark is clever – by different observers. This should prove to be an interesting solution to the problem of cognitive interpretations of patterns, and thus make a more objective pattern profitability research possible.

This thesis aims to answer the following question, “Is it possible to construct a mechanical trading system (based on fuzzy logic), which will be able to perform admirably on the BSM?” Due to time and resource constraints, we limit our analysis to a self-optimizing moving average and the infamous head-and-shoulders pattern. Our results are benchmarked to the returns implied by the Efficient Market Hypothesis (EMH). We expect TA to be profitable, thus rejecting the weak-form efficiency of the EMH for the BSM.

The rest of the paper is structured in the following way: we first present a literature overview, covering the controversy of TA in finance. A mathematical representation of the selected rules and fuzzy theory follow in subsequent sections. The fourth section presents the compiled model. We then report the results and address the reliability and data snooping issues. Suggestions for further research conclude the paper.

I. Literature Review

TA, or analysis of past prices, is very widespread today. Since the Dow Theory (the forefather of TA), which was formulated almost a hundred years ago, the idea that past prices can help predict future movements was one of the most discussed and applied theories (King, 1938). Recently, TA has come to the forefront of academic analysis, mostly due to perceived anomalies in the dominant financial philosophy – the EMH.

Technical analysis

In 1884, Charles Dow published his first moving average indicator. The intention was to allow public investors better visualization of general stock market movements (Befumo and Schay, 2006). Dow's later publications on the moving averages in his Wall Street Journal between 1899 and 1902 were subsequently summarized by S.A. Nelson (1903). The main issues were the presence of trends in stock prices, which could play a secondary role in investment decisions (the primary was value). Dow theorists (the most famous under studies of Dow Theory) correctly identified many reversals in market indices, making a fortune for the people who followed their advice, and spawning widespread interest in trends (Russell, c.1999).

Today, Dow Theory, and the more general TA, is widely used for both confirmation purposes (which is close to its initial function) and as a stand-alone trading model. The latter, albeit only done by a relatively small group of practitioners, uses TA as an independent predictive tool. The traders simply follow the trends with no interest in what the underlying stock is. This approach, as reported by Covel (2004), suggests informal evidence of profitability.

In the former case, traders use various rules to align a portfolio both with the trend and the intrinsic value. Value, which is the primary driver for prices, comes from a philosophy which emerged parallel to the Dow Theory, and which we today refer to as the fundamental analysis (FA). The idea is that a consistent link exists between share value and some outside factors, such as the state of the economy and the market segment. By translating these outside factors into cash flow projections, or other indicators, one could define a fair value for a stock (for an overview, see Oliveira, 2003).

Efficient Market Hypothesis

As these two philosophies were being investigated, and traders were using a mixture of the two, another powerful philosophy emerged – EMH (Fama, 1970), which effectively denied the right to life to any prediction model.

The idea of EMH is simple, yet powerful. It assumes that many independent rational profit-maximizing investors react to random new information by adjusting the prices of securities via buying and selling. If someone deviates from intrinsic value, arbitrageurs quickly seize the opportunity, and return the market to equilibrium. Therefore, past information, which incorporates the known information and the arbitrageur corrections, does not yield predictive power (weak-form efficiency). Recent public information appears at random, thus too not giving room for prediction (semi-strong form efficiency). Finally, insider information may only give limited excess returns, but, if measured by risk-adjusted returns, yields no abnormal profits (strong-form efficiency). Therefore, the market should be inexploitable, or efficient.

The framework of efficiency is confirmed via the Capital Asset Pricing Model (Sharpe, 1964; Lintner, 1965; Mossin, 1966), and the Arbitrage Pricing Model (Ross, 1976) which are almost universally used today. These models are used to set fair values for stocks, i.e. with no arbitrage opportunities possible, by using the fundamental links developed by FA proponents. A wealth of positive test results ensured a strong foothold for the EMH since the 70s, making it the supreme theoretical umbrella for all modern financial models (Fama, 1991).

Anomalies in EMH

People have, however, been shown to be overconfident in assessing their capabilities (Tversky and Kahneman, 1974). Despite research, traders continued using combinations of FA and TA, mainly due to the anecdotal evidence of success, and in light of various empirical criticisms of the EMH.

The EMH is attacked on some of its more general assumptions. First, investors may not be independent, i.e. they frequently follow advice. They may not be rational as defined by academicians, i.e. they may be buying when they should be selling (Olsen, 1997). Additionally, there are asymmetries of information, and evidence of trends, i.e. non-randomness of price movements. For overviews of these aspects, see Shostak (1997) and Lo and McKinlay (2002).

Academicians certainly defend the hypothesis. Malkiel (2003), for example, skillfully deals with the scattered evidence against the EMH (such as the January effect, market crashes). As for the more fundamental problems, Fama (1991) returns in episode two of the capital market efficiency argument to review, and refute, theoretical arguments.

Technical Analysis as Extension

Academic attempts at presenting a model which incorporates empirical anomalies in EMH conceived behavioral finance – a field of study which introduces psychology to standard finance. Behavioral finance argues that in real life financiers do not necessarily behave rationally (as defined by the EMH). For a comprehensive summary of behavior anomalies underpinning behavioral finance, see Schiller (1997).

Basically, behavioral inconsistencies, e.g. overconfidence, anchoring, lead to bounded rationality. These bounds (which are assumed away in the EMH) force participants towards a short-term sub-optimal equilibrium, rather than to a static CAPM-like general equilibrium. One example is that an investor may not be disciplined enough to hold on to a correctly identified stock when the rest are selling, thus “following the crowd”. In these cases, the link between the fair value and the share price is broken, and the EMH-suggested general equilibrium will not be reached (at least in the immediate future). It is exactly in these cases that heuristic TA rules help traders exploit mass movement to their advantage, or at least to hedge against losses (Rode *et al*, 1995).

Behavioral finance thus blends easily with TA: behavioral finance explains possible anomalies in the EMH-predicted price movements, e.g. flawed assumptions, whereas TA lends the tools to test evidence of such deviations. In this respect, TA is also a powerful auxiliary tool for traders, since it allows accounting for short-term (or even medium-term, e.g. the Enron case) irrational behavior of participants (Covel, 2004). In summary, by showing that significant abnormal returns can be gained by sole analysis of past prices, one can show that the market in question may be exploited to gain above-average returns by active portfolio management (*vis-à-vis* a buy-and-hold strategy).

Models

The testing of TA is, however, fairly difficult due to the cognitive nature of TA definitions, i.e. lack of consensus on definitions of tools. So far, the main ways of testing TA are either to assign constant definitions to tools, or to restate them over time, e.g. change sensitivity of the moving average (re-parameterize the rule). The controversy is due to lack of general theory to explain the validity of TA rules. Therefore, TA tests are frequently criticized as mere “intelligent data fitting” (Malkiel, 2003). Though refuted by a majority of academicians, TA models have, however, been implemented by a number of high caliber financial institutions (Feldman and Treleaven, 2004, p. 198).

The BSM has been analyzed by using both the static and dynamic definitions. Zaicevs (2003) used simple moving average crossovers, and Kukins and Strupka (2004) used constant value filters. Both papers use simple models with a number of possible values for rules, and contrast each to the EMH-suggested return. These papers mostly suggest that transaction costs do not allow significant abnormal returns.

A more comprehensive approach of using artificial intelligence to optimizing the rules was also investigated. Januškevičius (2003) used neural networks (on neural networks, see Azoff, 1994) to analyze VSE data, yielding favorable results for TA. Genetic algorithms (see Allen and Karjalainen, 1999) were used by Mihailov and Linowski (2001), and Arslanov and Kolosovska (2004). The results were mostly mixed, with some indication of inefficiency in the market. Though one less than the other, both neural networks and genetic algorithms are, however, heavily criticized for curve fitting and lack of explanatory power of underlying logic (Feldman and Treleaven, 2004).

Finally, fuzzy logic is the third advanced method applied in portfolio management. To our knowledge, it has not been used on the BSM. In fact, fuzzy logic has only been analyzed in the context of finance in a limited number of research papers, e.g. Dourra and Siy (2001) and Dong and Zhou (2004) [1].

[1] For usages of fuzzy logic in non-finance related areas (which is where it originally came from), see, for example, Bodhe, Navghare, and Dharmadhikari (2004).

II. Technical Analysis in Equations

A vast number of rules were spawned in line with recent interest in TA. Classifications are numerous, and can be found in such TA encyclopedias as Murphy (1999), and Hardy (1978). For the purposes of this work, the head-and-shoulders pattern and a self-optimizing moving average were used. These patterns are among the most frequently mentioned both by practitioners and academicians.

Moving Average

There are a number of moving average (MA) techniques, including e.g. exponential smoothening, and multiple crossovers. The results of simple crossovers on the BSM, for example, are discussed by Zaicevs (2003).

It is difficult to argue for the use of one or the other moving average. To add new information to research on the BSM, a self-optimizing moving average was chosen. The initial version was taken from Patel (1998, p. 28-29), which was equipped with price (parameter *p*) and time (parameter *q*) filters to improve performance during whipsaws. Parameter *p* was set in accordance with Patel (1998, p.12). Due to lack of theoretical knowledge on *q*, a number of different values were used.

To characterize a buy or a sell signal using fuzzy logic, two parameters for MA were calculated:

$$MA1: \quad Advance = t_T - t_{T-1}; \tag{1}$$

$$MA2: \quad \Pr ice = \frac{P_{t_T} - P_{t_{T-1}}}{P_{t_{T-1}}}, \text{ where} \tag{2}$$

T denotes an event – occurrence of signal;

(T-1) denotes an event prior to T, i.e. previous signal.

These parameters summarize the slope of the line from one signal to the next. MA1 calculates the time difference between two adjacent signals, and MA2 calculates the relative price change. Therefore, MA1 shows whether the signals have occurred after a long idle interval, e.g. the signal has been accumulating for a significant amount of time and is, therefore, more trustworthy. MA2, on the other hand, shows how far the price moved from the previous signal in the respective direction.

Head-and-Shoulders

A head-and-shoulders pattern (HS) is a combination of peaks and troughs which marks the reversal in price movements, i.e. from descending to ascending, and *vice-versa*. As indicated by the name of the pattern, it forms a general high (the head) with two sub-maximums on either side (shoulders). These highs and lows are denoted extremes (E). The pattern has been well known for decades, and is used extensively both as a primary and a secondary tool.

Initial research by Levy (1971) showed no excess profit for the patterns. Later works, which improved mathematical definitions of the HS, contradicted his findings (Brock, Lakonishok and LeBaron, 1992; Olser and Chang, 1995).

There are a number of complications with defining a mathematical recognition of this pattern. First, defining, and thus spotting, the HS pattern is partly art – the same set of extremes may be classified as HS by one trader, and not be classified as such by another due to simple differences in perception of the HS among these traders. On the research side, there is also the question of scale of search algorithm, i.e. tick-by-tick analysis (Dempster and Jones, 1998) or prior smoothening (Lo *et al* 2000).

Smoothening

We follow the smoothening path, since it allows defining of large-scale HS, i.e. patterns that are formed over weeks, months, and so on. As such, smoothening is required to eliminate noise in the data. A number of smoothening techniques are discussed in relation to price data. Different types of local smoothening are discussed by, for example, Loader (2004, p.18-22).

A variation of the simple kernel technique was used. Only past data was taken for the neighborhood estimate since our intention was to build a trading tool, which suggests no use of future data. The following kernel was applied:

$$K(t^*-t) = \frac{e^{-(t^*-t)^2/2h^2}}{h\sqrt{2\pi}}, \text{ where} \tag{3}$$

t* is the date at which P is estimated;

h is a nonnegative parameter (discussed further).

To calculate the weighted estimate of the price ($P_{t^*}^{Est}$), the weights were scaled and multiplied by the corresponding price values (also called the Nadaraya-Watson kernel):

$$P_{t*}^{Est} = \frac{\sum_{t=1}^{all}[K(t*-t)P_t]}{\sum_{t=1}^{T}K(t*-t)}. \qquad (4)$$

Therefore, for each t a weighted price P_{t*}^{Est} is estimated by applying the scaling in (4). Since t from the past is only taken from past data, in effect a moving average-type smoothening is used. Though this is not authentic kernel estimation, it avoids the look-ahead bias when smoothening the data.

Choosing parameter h (bandwidth), which controls the size of the local neighborhood, is important. The bandwidth is simply a parameter which measures how close the smoothened line is to the real data. If h is small, the averaging result is close to the initial price data. If h is large, then the power of the denominator is diminished, and smoothening becomes more like a flat line. Lo *et al* (2000, p. 1714) suggest a cross-validation technique. We omit this procedure due to time and resource constraints, and instead perform a number of tests with various h to support the robustness of the system.

The above smoothening process provides discrete estimates of a smooth price series. Previous works fitted a smoothened line to a fixed rolling window (e.g. Lo *et al*, 2000). We fit the regression for the whole data instead of fitting to a rolling window of observations, since we want to identify patterns that complete in any length of time (not only within the rolling windows).

Identification of extremes was done in two stages. First, extremes were identified in the set of smoothened data. This was done by a simple search for changes in the slope of the smoothened line.

$$\begin{aligned} &P_{t-1}^{E} < P_{t}^{E} \ \textit{and} \ P_{t}^{E} > P_{t+1}^{E} \ \textit{for} \ \max; \\ &P_{t-1}^{E} > P_{t}^{E} \ \textit{and} \ P_{t}^{E} < P_{t+1}^{E} \ \textit{for} \ \min. \end{aligned} \qquad (5)$$

The second stage was assigning status of an extreme to the respective maximum or minimum in the original price series. In this paper, the notation for a maximum is H, and for the minimum, the notation is L.

HS Identification

As mentioned above, the basic definition of a HS2 is that the middle peak (head, H2) is higher than the two adjacent ones (shoulders, H1 and H3). Two troughs are located between the three peaks: L1 and L2. The line going through L1 and L2 is called the neckline, and through the H1 and H3 – the shoulder line (Fig. 1).

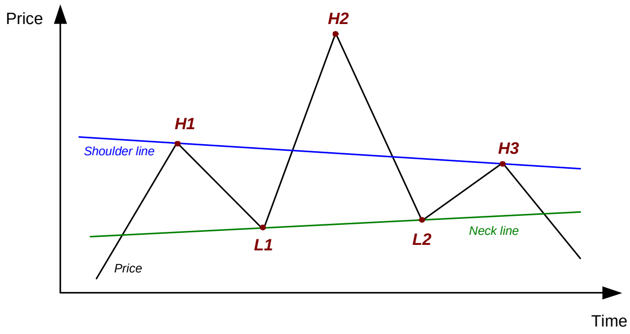

Fig. 1. A normal head-and-shoulders pattern
Source: Created by authors

In order for the pattern to be "recognizable", a number of other restrictions are placed. Savin, Weller, Zvingelis (2003, p. 7-9) produced a respectable set, which we narrowed down to five, and expressed as equations, not constraints (Appendix A provides more details on derivation of these parameters):

$$
\begin{aligned}
&HS1: \quad Shoulder = \frac{H1}{ShAverage} - 1, \quad where\ ShAverage = \frac{H1 + H3}{2}; \\
&HS2: \quad Neck = \frac{L1}{NeckAverage} - 1, \quad where\ NeckAverage = \frac{L1 + L2}{2}; \\
&HS3: \quad Head = \frac{H2}{ShAverage} - 1; \\
&HS4: \quad Body = \frac{ShAverage}{NeckAverage} - 1; \\
&HS5: \quad Skew = \max_i (E_{i+1} - E_i - E^*), \quad where\ E^* = \frac{(E_1 + E_5)}{5}, \quad i = 1..5.
\end{aligned}
\tag{6}
$$

The first and second equations give a feel for the slope of the shoulder line and the neckline, as well as the spread of individual highs and lows. The third equation specifies how well the head is represented (as measured against the shoulder line), and the fourth one yields the broadness of the body of the pattern. Finally, the fifth element measures the vertical asymmetry of the pattern. Thus, we have "described" the pattern – by looking at these five parameters, one

[2] There are two types of HS. We describe the normal one, and leave out a description of inverse HS. As suggested by its name, the characteristics of inverse HS are inversely related to the normal one. Though we do not describe it, we still use it in the model.

can see if it is skewed to the right, has heavy shoulders, or, for example, a small head. This will (as in case of MA) help identify the patterns more precisely once we merge it with fuzzy logic.

A final note is on when to act upon HS, i.e. when to assume the pattern was actually spotted in the price series. Previous papers assigned an arbitrary number of days following the formation for the investor to “see” the HS. This was assumed to be the day the investor acted upon the HS.

We took a different perspective, based on the frequent remarks of TA practitioners about the confirmation principle of the neckline. More specifically, a trade is conducted based on HS when the neckline *penetrates the price series*. For the normal HS (we are selling stock), the neckline should be penetrated by the price series from above. If it does not penetrate the price series, e.g. a new minimum occurred prior to prices falling below the neckline projection, then HS is considered to be incomplete, and therefore disregarded (Murphy, 1999).

To find this sixth point of HS, the functional form of the neckline was derived by using the L1 and L2. Thus, the algorithm searched for:

$$P_{L_2} + \frac{P_{L_2} - P_{L_1}}{t_{L_2} - t_{L_1}} \cdot t \geq P_t \text{, where} \tag{7}$$

P_L is the respective (according to number) low.

Basically, as soon as a HS pattern was identified, the neckline was constructed and the program verified that the neckline penetrated the price before a new extreme was recorded. If this condition was satisfied, the P_t was recorded. This was the price at which the trader was assumed to act upon the HS. If the condition was not satisfied, the pattern was disregarded.

III. Fuzzy Logic

Having defined the characteristics of the MA and HS pattern, a system was constructed to interpret them. For this, interpretation of parameters needed to be defined. One could, for example, have assigned each combination a weight, e.g. if both HS1 and HS2 = 0, HS3 = 100%, HS4 = 100% and HS5 = 1 (which constitutes a nice symmetric HS), investing 100% of allocated funds. But, given the range and continuous nature of the five variables, it was impractical to do so.

Intervals could have solved the problem, but the difficulty was that the exact boundaries were unknown. Suppose one set the boundary value at $|HS1| \leq 50\%$, i.e. we did not consider that a HS pattern occurred if the shoulders were more than 50% away from their average. In that case, a value of 51%, even though very close to the true value, will be disregarded. Given that the TA proponents do not have a unified boundary, e.g. some might suggest 50%, and others use a value as low as 4%, setting such strict boundaries was not smart. This is what academicians refer to as the problem of cognitive definitions – there is an *interval* of correct values for a boundary, thus one needs the boundary to be smooth.

Fuzzy sets

Fuzzy logic (FL), introduced by Zadeh (1965), helps solve this problem. It extends the Boolean "either… or…" logic by allowing various "membership functions" to intersect, thus making the decision – agreement or disagreement with the statement – flow continuously from one state to the other.

The vagueness of definitions allows "tolerance for imprecision which can be exploited to achieve tractability, robustness, […] and better rapport with reality" Zadeh (1999, p. 109). In other words, we should benefit from the degrees of truth and falsehood which fuzzy logic allows – having a smoothening of the boundary conditions allows better mimicking of real-life trading decisions, as well as a better description of what we *believe* is a worthy HS and MA.

Model Derivation

We first present the complete procedure in terms of the MA. As noted above, the MA gives a buy or sell signal based on a specific event: when the prices cross MA from below, we buy; in reverse, we sell. The next question is – buy or sell how much? For a portfolio which is optimized once, this is not a very serious question. Kukins and Strupka (2004), for example, assigned equal weights to the chosen stocks. For genetic algorithms, portfolio re-optimization was the key to determining weights in portfolios (Arslanov and Kolosovska, 2004), i.e. the computer decided based on its own algorithm.

Fuzzy MA Model

Fuzzy logic proposes another way of dealing with the issue of portfolio composition. It allows thinking in terms of *how confident* we are about the signal. For these purposes, we took the two parameters presented in the previous section, MA1 and MA2, and proposed "signal powers" based on the combined value of parameters. We could, for example, propose a decision matrix similar to the one in Table 1.

Table 1. Decision Matrix for the MA

		MA1 (Advance)		
MA2 (Price)		*Small*	*Moderate*	*Long*
	Small	$W_{Small,Small}$	$W_{Moderate,Small}$	$W_{Long,Small}$
	Medium	$W_{Small,Medium}$	$S_{Moderate,Medium}$	$S_{Long,Medium}$
	Large	$S_{Small,Large}$	$S_{Moderate,Large}$	$S_{Long,Large}$

Source: Created by authors

In the table, *W* refers to a weak, unconvincing signal, i.e. where doubts towards pessimism outweigh the positive uncertainty. *S* is exactly the opposite, where we are sure about the signal, or, in other words, the descriptions of the signal exactly matched what we believe to be a perfect penetration. The logic behind the average is as follows: if the trend is lengthy (Advance variable is high) and healthy (price change is stable), we buy confidently (*S* signals). Otherwise, we invest cautiously (*W* signals).

Here, fuzzy logic variables are first used. Assignment of the *W* and *S* to a combination of parameters is semi-arbitrary, i.e. based on scarce theoretical information available from TA encyclopedias.

The next immediate problem is quantifying the three states of each variable. Consider the MA1 variable – it can either be SMALL, MODERATE, or LONG. Adjectives are used to reinforce the idea that there are no strict boundaries for parameters. In other words, the problem

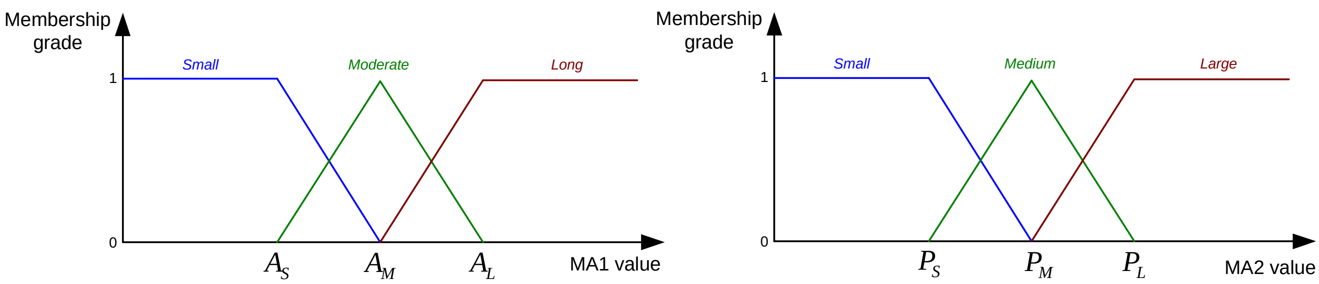

Fig. 2. Membership functions for MA1 (Advance) and MA2 (Price)
Source: Created by authors

is that we do not exactly know the values which strictly distinguish one from the other. To solve this problem, memberships were used.

The membership relation of the three descriptive statements about the MA1 and MA2 variables of the MA (triangular) are depicted in Fig. 2. Parameters A_S, A_M, and A_L, and P_S, P_M, and P_L stand for the centroid values of the function. These are the *perceived* boundaries, i.e. what we *think* the boundaries are between the three descriptive statements. Once we calculated MA1 and MA2, we can find their membership in one of the three states. For example

$$A_S < MA1 < A_M \text{ and } P_S < MA2 < P_M \tag{8}$$

i.e. both the MA1 and MA2 parameter belonged to two statements – SMALL and MODERATE, and SMALL and MEDIUM, respectively.

To find the degrees of membership in each state we use a simple mathematical description of the fuzzy functions. Table 2 illustrates the computational process:

Table 2. Computation of Memberships for the MA Parameters

MA1 (Advance)	$MA1 \le A_S$	$A_S < MA1 \le A_M$	$A_M < MA1 \le A_L$	$A_L < MA1$
M($MA1$)$_{Small}$	1	$\frac{A_M - MA1}{A_M - A_S}$	0	0
M($MA1$)$_{Medium}$	0	$1-\frac{A_M - MA1}{A_M - A_S}$	$\frac{A_L - MA1}{A_L - A_M}$	0
M($MA1$)$_{Large}$	0	0	$1-\frac{A_L - MA1}{A_L - A_M}$	1

MA2 (Price)	$MA2 \le P_S$	$P_S < MA2 \le P_M$	$P_M < MA2 \le P_L$	$P_L < MA2$
M($MA2$)$_{Small}$	1	$\frac{P_M - MA2}{P_M - P_S}$	0	0
M($MA2$)$_{Medium}$	0	$1-\frac{P_M - MA2}{P_M - P_S}$	$\frac{P_L - MA2}{P_L - P_M}$	0
M($MA2$)$_{Large}$	0	0	$1-\frac{P_L - MA2}{P_L - P_M}$	1

Source: Created by authors

The notation is similar to that described in the previous section, where the subscript denotes the state, e.g. SMALL. The M($MA2$)$_{Small}$ should thus be read as "membership of parameter MA2 to state SMALL".

Having identified the membership values, one can refer back to the decision matrix. Zadeh and Bellman, as qtd. in Ramik (2001, p. 109), showed that in calculating a string of fuzzy parameters, the minimum value should be used. More generally, one should define all the combinations of parameters (in this case a three-by-three matrix) and find the minimum of the two membership values assigned to it. This value is then assigned to either *W* or *S*, depending on how one has decided to interpret the combination.

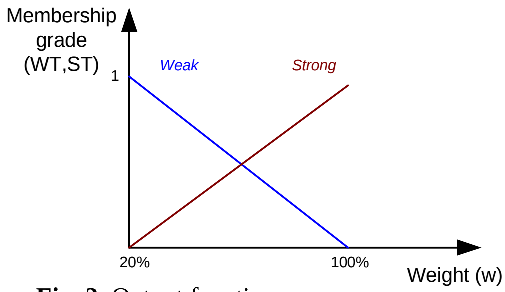

Fig. 3. Output function
Source: Created by authors

For the MA, the decision rules look in the following way:

$$\begin{aligned}
&\text{Rule1: } W_{Small,Small} = \min\,[M(MA1)_{Small};\ M(MA2)_{Small}] \neq 0;\\
&\text{Rule2: } W_{Smal,\ Medium} = \min\,[M(MA1)_{Small};\ M(MA2)_{Medium}] \neq 0;\\
&\text{Rule3: } S_{Small,Large} = \min\,[M(MA1)_{Small};\ M(MA2)_{Large}] = 0;\\
&\text{Rule4: } W_{Moderate,Small} = \min\,[M(MA1)_{Moderate};\ M(MA2)_{Small}] \neq 0;\\
&\text{Rule5: } S_{Moderate,Medium} = \min\,[M(MA1)_{Moderate};\ M(MA2)_{Medium}] \neq 0;\\
&\text{Rule6: } S_{Moderate,Large} = \min\,[M(MA1)_{Moderate};\ M(MA2)_{Large}] = 0;\\
&\text{Rule7: } W_{Long,Small} = \min\,[M(MA1)_{Long};\ M(MA2)_{Small}] = 0;\\
&\text{Rule8: } S_{Long,Medium} = \min\,[M(MA1)_{Long};\ M(MA2)_{Medium}] = 0;\\
&\text{Rule9: } S_{Long,Large} = \min\,[M(MA1)_{Long};\ M(MA2)_{Large}] = 0.
\end{aligned} \tag{9}$$

For inequalities in (8), the $M(MA1)_{Long}$ and $M(MA2)_{Large}$ are zero, and therefore rules 3, 6, 7, 8, 9 yield a zero value, and only four rules fire non-zero values (rules 1, 2, 4, 5).

We now have a set of values for *W* and *S* (for example above, one value for *S* and three for *W*). A common way to summarize these values is the Root-Sum-Square method (Kaehler, 1998).

$$\begin{aligned}
WT &= \sqrt{\sum W^2_{\text{Small,Small}} + W^2_{\text{Small,Medium}} + W^2_{\text{Moderate,Small}} + W^2_{\text{Long,Small}}};\\
ST &= \sqrt{\sum S^2_{\text{Small, Large}} + S^2_{\text{Moderate, Medium}} + S^2_{\text{Moderate, Large}} + S^2_{\text{Long,Medium}} + S^2_{\text{Long,Large}}}.
\end{aligned} \tag{10}$$

This procedure yields set-theory-type averages: *WT* and *ST*, which are fuzzy outputs. Alternatively put, we have a feeling for how weakly convinced and strongly convinced (both at the same time!) we are. These values, however, need to be merged to arrive at a distinct weight. For that, an output fuzzy function is specified, which “defuzzifies" the parameter (Fig. 3). There are many different ways to defuzzify this output (Mendel, 1995, p. 368-369), which are basically different ways of merging values. We use the popular “height defuzzification” procedure, which is simple in calculation and quite applicable to our needs. The defuzzification procedure (crisp output) is summarized by the following formula:

$$\frac{0{,}2 \cdot WT + 1 \cdot ST}{WT + ST} = w\,. \qquad (11)$$

The parameter *w* tells how much to invest in a stock (or, alternatively, how much to disinvest in it) as a percentage of what would have alternatively been invested if the model was binary. That is, suppose the policy was to invest 5,000 EUR in a stock based on the MA being breached by price line from below (binary stock model). Under the fuzzy logic system, we would invest 5,000 EUR multiplied by the *w* parameter. To see the coherence of the model to common sense, assume there is (as defined by the membership parameters) a beautiful solid penetration. That is, both parameters belong to states LARGE and MEDIUM, and LONG and MODERATE, respectively. This will fire only four rules (all of which are S signals), which turns to $ST = \sqrt{1^2 + 1^2 + 1^2 + 1^2}$. Substitute that in the output equation ($WT = 0$) to arrive at w = 100%. Thus, in extremes, fuzzy output converges on the crisp output, and at the same time yields by far more interpretable results in-between the two extremes.

Fuzzy HS Model

We now turn to the HS model. The HS pattern, the second part of the program, is described by five parameters (versus two for MA). This increases the facets of the decision, and makes it tricky to use matrices to show diversity of the decision power. Instead, a decision *tree* is constructed to fully reflect the possible combinations of the parameters (Appendix B). As evident from the tree, HS4 and HS5 have two states, while the other three parameters have 3 states. This brings the total number of rules to $3^3 \cdot 2^2 = 108$. Each rule is assigned an outcome, *W* or *S* (same logic as for MA matrix). The membership sets are depicted in Fig. 5 (Appendix C).

The diagrams implicitly show the calculation procedure of membership to each statement (we do not include the mathematical explanations, since they are easily derivable from the charts). The procedure is similar to the MA in that we fuzzified the parameters, processed them, and defuzzified using the same algorithm as in Fig. 3. We use the trapezoid functions in line with Dong and Zhou (2004).

IV. The Model

This section discusses more practical issues of the program algorithm. Input data, along with the computation process and means of comparison are explained. The software which was programmed for this paper is also briefly presented.

General Algorithm

The trading program built for the purposes of this paper works in the following way. First, adjusted input data on the stocks is fed in. The program then constructed the required instruments, the MA and HS, and recorded for each stock the date, type of signal (buy or sell) and the fuzzy output weight (*w*). This was based on inputs that can be easily changed within the system. The next step was to find the value to which these weights were applied, i.e. a limit on the identified position (explained in the next section). This gave the investment (disinvestment) value in monetary terms. The program then tracked the daily change in portfolio value (the stocks and the risk-free asset). These changes were used to calculate the annualized returns and standard deviations of the portfolios. Thus, the program allowed easy input of parameters, and produced the outputs required to either confirm or reject the hypothesis of this thesis. Appendix D provides a few snapshots of the program**.**

To be able to compare the results of the EMH and TA, the results were presented using risk-adjusted returns. We used the Sharpe ratio (Sharpe, 1975) since there was no significant evidence that good diversification was possible within the BSM:

$$\text{Sharpe ratio} = \frac{r - rf}{\sigma}, \textit{ where} \tag{12}$$

r is the annual return;

σ is the annual standard deviation of return;

rf is the risk free rate.

Transaction costs were also accounted for as a separate parameter (parameter *tc*), while a risk-free rate was applied to cash not invested in stocks (parameter *rf*).

As Malkiel (2003) quoted himself, a “blindfolded chimpanzee” should pick the stock to represent a buy-and-hold portfolio. We used the buy-and-hold strategy on BALTIX (the BSM index) as a benchmark for this purpose (a description of the BALTIX can be found on the OMX web-site). By comparing the risk-adjusted returns of the model and the BALTIX, the dynamic portfolio returns could be measured against the EMH returns.

Limit calculation

One of the major difficulties is setting limits on positions. Limits are a simple constraint which forces diversification of a portfolio since it does not allow holding too much of the same stock. A great number of techniques exist, most of which are discussed in practical application, not theoretical. We use percentage limits, as described in, for example, Sperandeo (1994, p. 4-8).

First, we had the initial portfolio, which is all cash. When the first signal was generated, the model calculated a percentage (*l*) of the total portfolio value ($V_{Portfolio}$) and used that figure as the limit on position (v), given that it did not exceed cash supplies:

$$v = V_{Portfolio} \times l\,. \tag{13}$$

This was the maximum allowed position, where as the actual amount of the funds to be invested was $v \times w = p$ (as discussed above). This procedure applied to the buy signal; for a sell signal, *p* was substituted for the dollar amount of the position that needed to be liquidated.

The third step suggested calculation of new portfolio value, i.e. the sum of open positions and cash. For the latter, a risk-free rate was applied. This was computed daily, but the actual additional transfer of funds to the account occurred on a monthly basis (in line with reality). Daily returns were recorded for the whole portfolio (stock and cash). The procedure was repeated when a new signal was generated (Fig. 7, Appendix E).

One interesting aspect is the possibility of the same signals in a row, e.g. two buy signals without a sell in the middle. Unfortunately, there are no theories regarding this nuance, hence no reference to adjustments can be made. In our case, four decision trees were constructed for the MA module: a case where (1) a buy signal is preceded by another buy signal and (2) if it is preceded by a sell signal (this is the matrix in Table 1). Similarly, two types of trees were constructed for a sell signal: if a sell signal is preceded by a (3) buy signal and by (4) a sell signal.

For HS, the process could not have been dealt with in such a straightforward way due to the vast number of possibilities. For the purposes of reasonable simplicity, each HS signal was viewed without regard to the previous one.

Data

Previous models suggested using stocks that conform to a certain level of liquidity (Arslanov and Kolosovska, 2004, p.16-17). These constraints effectively included only the “blue-chip” main-list companies. Since there were no well-documented arguments supporting low liquidity of the secondary list, we used all the stocks from primary and secondary lists.

The raw data (which is downloadable from the stock exchange website) was adjusted for splits. No adjustments were made for dividends due to time and resource constraints (the adjustments need to be made manually, and general dividend data is not available). The dividend adjustment problem should not, however, influence the end results drastically, since for both TA and EMH data no dividends were recorded. Finally, one company was dropped due to lack of information regarding its price series, i.e. standard adjustment did not apply.

This left a database of 69 (Appendix F) companies (12 on RSE, 15 on TSE, and 42 on VSE). For each company, historic close price series (in EUR) were acquired for the period starting 01/01/2000 and ending 01/01/2006. Some stocks were listed later than the start date, and there were breaks in trading for others. The trading program adjusted for both effects.

V. Results and Discussion

To simulate the results, a range of values was suggested for the main inputs (see Appendix G for a full list of parameters), and each was iterated to find the risk-adjusted returns of the model under different parameter combinations (Table 3).

Table 3. Ranges of the Parameters

Parameter	Range	Step
h	[5;40]	1
q	[2;7]	1
n	[10;60]	10
l	[5;10]	1
tc	[0.3;0.8]	0.1

Source: Created by authors

The first parameter is the smoothening extent for HS. The second and the third are the MA parameters – required penetration (in days) and length of the initial moving average (also in days), respectively. The last two are general parameters – limit size (5 denotes 5%) and transaction costs (0.3 denotes 0.3%).

The number of generated results was 163 296 observations, split equally among the two tools. Primarily, the large number was due to the many combinations of parameters. Additionally, data was generated for different periods within the sample; more specifically, all possible intervals of 2000 – 2006 with a minimum step of one year (altogether, 21 intervals) were generated. Finally, there was the issue of investment order, i.e. which signal to interpret first if on a given day a number of decisions needs to be made (e.g. three separate buy signals). To account for this, data were generated for each interval and combination of parameters *three times*, specifying the order of investment at random for each of the three runs.

Before proceeding with a discussion of the performance of individual tools, it is worth looking at the performance of the benchmark (the BALTIX). Fig. 8 (Fig.8 to Fig.17 can be found in Appendix H) shows how the Sharpe ratio and the annualized returns changed over various intervals of time[3]. Apart from the *tc*, which had a very limited effect on returns, none of the other parameters affected these figures. Thus, by looking at them, one can see how the market developed over time.

Overall, the returns for the market were greater within the last few years. This is in line with what was written in the introductory part of the paper: the market has been far more dynamic. The Sharpe ratio follows the returns quite closely, though in the latter periods the Sharpe ratio levels out. This should be due to the increased volatility of returns in recent years (2004-2005).

Head-and-Shoulders

The HS pattern is easier to interpret (than MA) due to the smaller number of dimensions. Here, there is only one “main” parameter – *h*. The other two parameters (*l* and *tc*) are, in a way, secondary and can be easily summarized by, for example, an average. Fig. 9 summarizes the performance of the HS pattern within various intervals. The vertical axis of the first graph shows the fraction of total observations where the HS pattern outperformed EMH-suggested returns (as measured by the Sharpe ratio), whereas the second graph shows annualized returns. An interesting observation is that the pattern performed well only over long intervals of time. Furthermore, though the HS returns rarely outperformed the EMH, the Sharpe ratio seems to do

[3] It is worth noting that there is a link between these graphs. Since the Sharpe ratio essentially returns over the standard deviations, one could implicitly see the development of the standard deviation by comparing the Sharpe value and the value of return.

better. This could suggest that the HS pattern won against the EMH in terms of variance, i.e. HS managed the variability of returns better.

Fig. 10 illustrates how the performance of the HS changed if one only counts the number of times the HS outperformed the EMH by a certain percentage. The results were fairly predictable, i.e. they declined as the required “performance margin” increased. The first graph, for example, shows how many times the HS pattern outperformed the EMH by more than 10%.

Next, we looked at how changes in parameter *h* affected the results (Fig. 11). Interestingly, there is no clear trend. However, it seems that smaller values (patterns are found more frequently) produced better returns. This suggests that dynamic asset management should be a better idea than long-term investments. On the other hand, performance linked with extremely high results is also fairly good.

Finally, changes of parameter *l* and parameter *tc* yielded predictable results (Fig. 12 and Fig. 13). Though academic literature frequently states that transaction costs should not allow active portfolio management to outperform the buy-and-hold strategy, it seems that these costs have had little effect on performance of the model. Limits had a greater impact, though in some cases the results opposed the diversification principle, i.e. an increase in limits actually leads to an increase in returns. These “anomalies” were present mostly for small and large values of *h*, which reinforces the idea that extreme values of *h* performed better than the average.

In summary, three interesting observations emerge from the discussion above in relation to research on technical rules on the BSM:

- Indirect evidence exists that the HS is capable of managing variability of returns.
- Both the relatively short- and long-term approaches to using the HS pattern yield better results than use of an average value. This notion needs to be investigated further, though, since small changes in *h* dramatically affect returns, i.e. the link is unstable.
- Limits, i.e. level of diversification, have a stronger influence on the HS pattern than transaction costs. This is interesting, since academicians frequently address transaction costs as the main foe of active portfolio management, and rarely speak of the effect of diversification in relation to the TA.

Moving average

The moving average has two major variables: *q* and *n*. The general results are by far more optimistic than those produced by the HS pattern (Fig. 14).

The model outperformed the market significantly in most cases (this can be seen both visually from the graphs and from the t-statistics provided in Appendix I). It performed poorly in the initial periods (year 2001, 2002), which is in line with previous findings by Zaicevs (2003) and Kukins and Strupka (2004). The change from superb performance in terms of returns to poorer performance in terms of the Sharpe ratio in recent years once again confirmed the higher volatility of the latter periods.

"Marginal" performances were also quite good (Fig. 15), which was further supported by decisive rejection of the hypothesis that the difference of MA and EMH returns is negative (see Appendix I).

In terms of the MA-specific parameters – *q* and *n,* the picture is less clear (Fig. 16)**.** From the graphs, it seems that short-term moving averages have performed better than long-term ones. Though the graphs show almost the same dimensions, the results are somewhat contradictory. These need not, however, be true: perhaps, the issue is that the more dynamic moving averages have indeed performed admirably, but so did the EMH. This logic inevitably confirms the primary rule for the moving averages – that a short-term MA captures the trend quicker. However, TA proponents frequently note that the sensitive averages leads to losses in whipsaw markets. Combining the superb performance of the moving average, and the two graphs below, an interesting observation emerges – the BSM market did not exhibit whipsaws, i.e. short-term fluctuations. This observation is also in favor of TA.

Finally, the analysis of costs and limits suggests similar results to the HS pattern (Fig. 17). With increased limits the number of times MA outperformed EMH decreases, whereas transactions costs seem to have little effect on returns.

In summary, MA has outperformed EMH significantly. One highly intriguing finding is that the short-term average outperformed long-term moving averages, which suggests *stable* trends.

Significance

The testing of TA rules has been much criticized due to data snooping issues. Generally, data snooping is over fitting of parameters in the model (or model family) to the actual price output, which seemingly yields a high predictive power, but is prone to poor forecasting if the underlying relations change a little (neural networks and genetic algorithms are often criticized on this). Another source for data snooping is the result of reuse of data, i.e. applying massive amounts of rules to the same data period. In this case, any superiority could be achieved by mere chance.

Academicians suggest re-testing data on out-of-sample data to see how the rules perform elsewhere. Park and Irwin (2005) have, for example, replicated a previous study in the futures market to show that significant positive returns in 1978-1984 have declined over time, and disappear for 1985-2003.

Standard t-tests are included in Appendix I to confirm significance of the findings. Along with density functions, we report a summary table which shows how the Sharpe value and returns have significantly outperformed the respective EMH ratio. Technically speaking, we were testing the null hypothesis that the difference in TA returns and EMH returns was equal to zero. This was done using standard statistical software STATA.

Model characteristics

One of the limitations of the model is the large number of parameters that need to be set (also mentioned in Feldman and Treleavan, 2004). The model is not among the most extensive, yet even at this stage many inputs need to be considered. Along with the general inputs discussed, there is also the general cognitive description of the MA and HS which needs to be set, i.e. which combinations of parameters should be interpreted as a strong signal, and which as a weak signal. For HS alone, this requires defining 108 combinations. Altogether, almost 200 parameters need to be fed into the model. However, most of the parameters have to be set only once, and are easily interpretable, which somewhat mitigates the problem. The number of active parameters, i.e. those that were subject to various interpretations, narrows down to around a dozen.

The positive side of the model is its transparency (as set against, for example, neural networks and genetic algorithms). Moreover, the effects of change in these models are predictable, thus giving the traders a tool for very concrete definitions of their beliefs about TA. In other words, fuzzy modeling allowed shifting some of the technical chart-reading from professional judgment to computer calculations, giving more robust, and at the same time sufficiently flexible, results. In summary, the number of parameters can actually be of benefit to traders, since they will be able to define their needs and beliefs in a more precise way.

Conclusions

This paper has shown the validity and value of TA on the BSM. This was achieved via fuzzy logic, which allowed cognitive thinking. Moreover, we have proved that active portfolio management – based on two technical rules – managed to (at least partially) outperform the passive EMH-based strategy. This shows that the BSM is not, at least to this point, efficient, and TA can be used as an auxiliary, or even stand-alone, tool.

The main finding is that the model has been able to achieve significant returns. Though for one less than for the other, both rules seem to perform quite well. Furthermore, for both the MA and the HS, a change in parameters yielded logical results, which suggested stability of the model, e.g. an increase in limits leads to lower returns. In terms of result interpretation, a number of curious trends emerged, which suggest more research is needed to address the theoretical grounds for the validity of, for example, the HS. Its effect on managing volatility in returns, and non-linear relationship to the time-frame are all interesting aspects which have not been touched upon by the academic literature yet.

Finally, in parallel to the main research question, which shows that fuzzy logic is a valid field for further studies, we have also suggested a few other interesting theoretical notions. Specifically, we have augmented the theoretical knowledge on the HS pattern by introducing the "reaction point" and by systematizing the descriptive parameters. Though we have not checked if these innovations have improved trading results, we have certainly merged theory and practice to a larger extent.

In summary, the paper concludes that the research design is valid and profitable, and further research based on the fuzzy model should be encouraged. Additionally, interesting extensions to HS could be investigated.

Suggestions for Further Research

Fuzzy logic is highly multi-dimensional, and therefore further research could be conducted in a number of directions. A large area for research is the model itself, which should be subject to modifications and optimization. Different types of fuzzy functions, e.g. S-shaped functions, could be tested. Optimization of boundary parameters, based on questionnaires of professional technical analysts, could be carried out. Other classes of technical rules could be considered, e.g. oscillators, which would take into account another part of trading which we did not study – the liquidity of the BSM. In other words, possible improvements to the fuzzy model could be investigated.

Another interesting dimension arises from the unexpected results within the sample. It seems that the six-year history of the market includes both stale periods (2001, 2002) and dynamic periods (2004, 2005). This is a perfect opportunity to test various combinations and parameters of rules in different market environments. Such research could help traders further understand relationships between the market "mood" and the tools they are using.

The HS pattern itself is an interesting topic for further discussion. Though it did not perform as well as the average, its curious role in volatility management should be looked into. This calls for volatility analysis using, for example, GARCH models. Additionally, the unexplained sensitivity of the HS to the h parameter should be investigated. Modification to the definitions of the HS proposed in this paper could also be a part of this study.

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Appendices

Appendix A. HS Parameters

In this appendix we show the reasoning for changing a number of constraints (mathematically and conceptually) from those produced in (to the best of our knowledge) the most recent paper on the topic (Savin, Weller, and Zvingelis, 2003). The motivation for these modifications is (1) decreased computation time, and (2) better descriptive power of the rules for purposes of fuzzy logic. For a full explication of rules, refer to their work.

Savin, Weller, and Zvingelis (2003) have modified the Lo *et al* (2000) version of what constitutes a head-and-shoulder pattern in a number of interesting ways. They introduce (based on work by Bulkowski, 1997) a measure for the head of the pattern, and rule out vertical asymmetries. Disregarding the basic definition of the HS, six rules are presented.

First, we change the specification for shoulder line and neckline (R4a and R5a). The original rules were (Es are the extremes, i.e. E_1 is the left shoulder):

R3a: $$\max_i \left| E_i - \overline{E} \right| \leq 0{,}04\overline{E},\ where\ \overline{E} = \frac{E_1 + E_5}{2},\ i = 1{,}5\,;$$

R4a: $$\max_i \left| E_i - \overline{E} \right| \leq 0{,}04\overline{E},\ where\ \overline{E} = \frac{E_2 + E_4}{2},\ i = 2{,}4\,.$$

For both equations, it can be shown that the absolute values are actually the same. Consider the value for R3a. If we simplify the two equations, we see that they hold by definition:

$$| E_1 - \frac{E_1 + E_5}{2} | = | E_5 - \frac{E_1 + E_5}{2} |;$$

$$| E_1 - E_5 | = | E_5 - E_1 | .$$

We therefore simply need to know the difference between one of the extremes and the average to identify the deviations of (in this case) peaks from the shoulder line. Moreover, the sign of the difference will reveal the slope of the shoulder line. By making the necessary changes, and expressing the difference in percentages, we receive HS1 and HS2.

A similar simplification is made on the expression for skew (R9), since

$$X^* = \frac{\sum_{i=1}^{4} (X_{i+1}^* - X_i^*)}{4} = \frac{(X_5 - X_1)}{4}\,.$$

Finally, we modified the relationships of the head to the shoulders. First, we merged R6 and R7 into HS3 since we work with equations, not constraints. Secondly, we substitute R8 with HS4. We drop R8 since it is closely correlates to R7, i.e. neckline never goes above the shoulder line by definition; we introduce HS4 to observe how well the body is defined. It is true that combination of R5, R6, and R7 implicitly define the body of the HS; HS3, however, summarizes the notion in a more meaningful manner.

Appendix B. Decision Tree

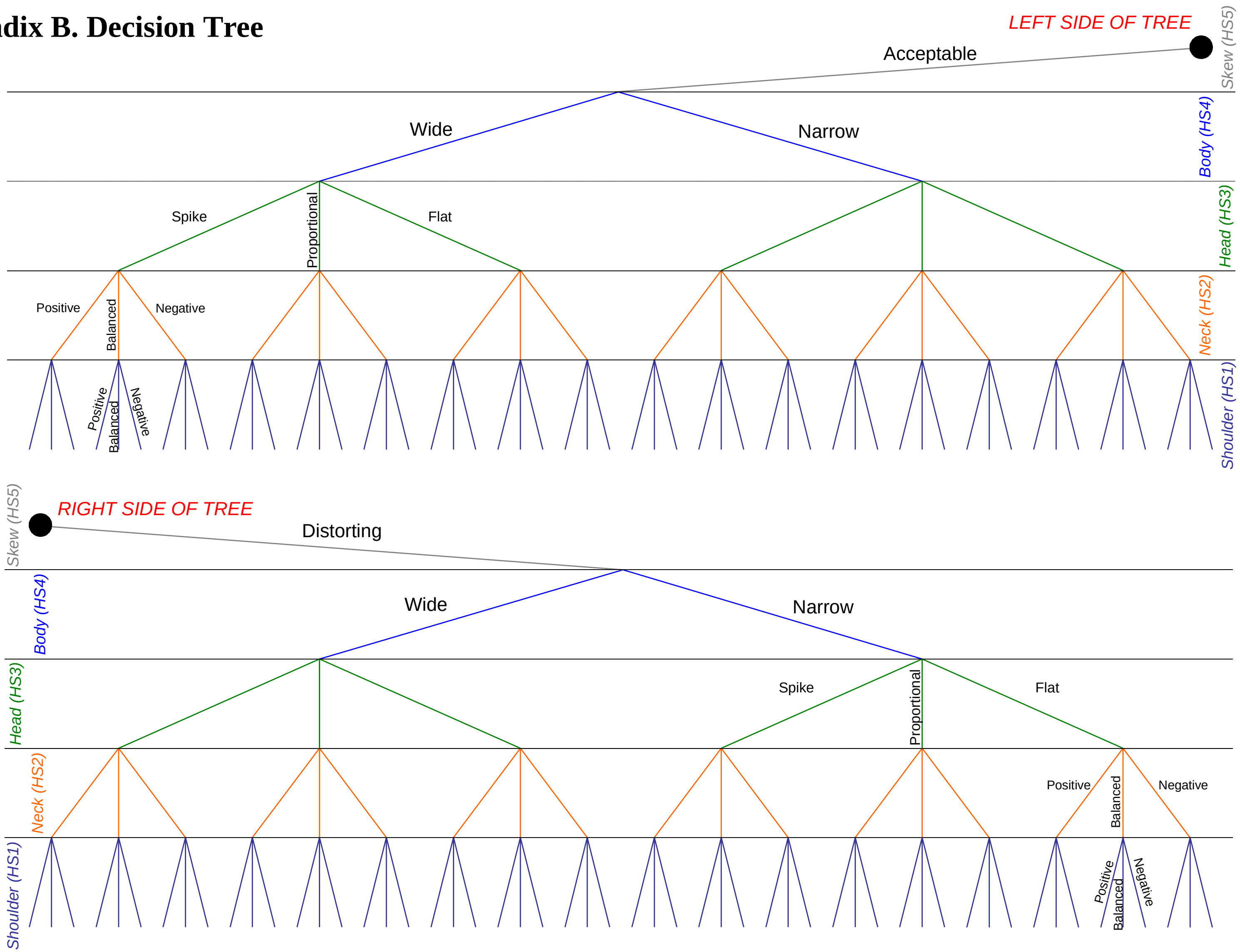

Fig. 4. Decision tree for HS pattern
Source: Created by authors.

Appendix C. HS membership functions (graphic representation)

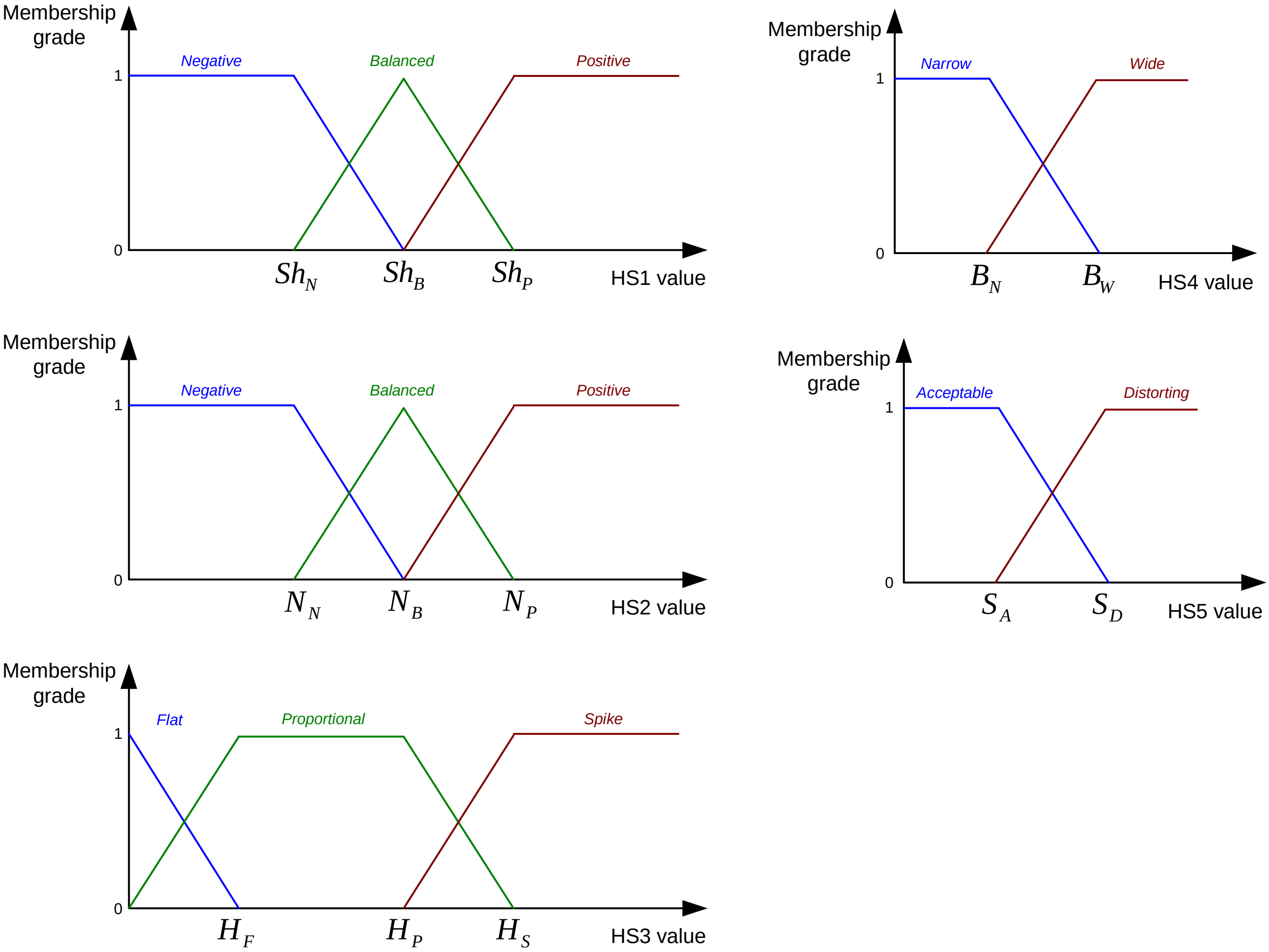

Fig. 5. Fuzzification of the HS pattern
Source: Created by authors

Appendix D. Snapshots of the trading program

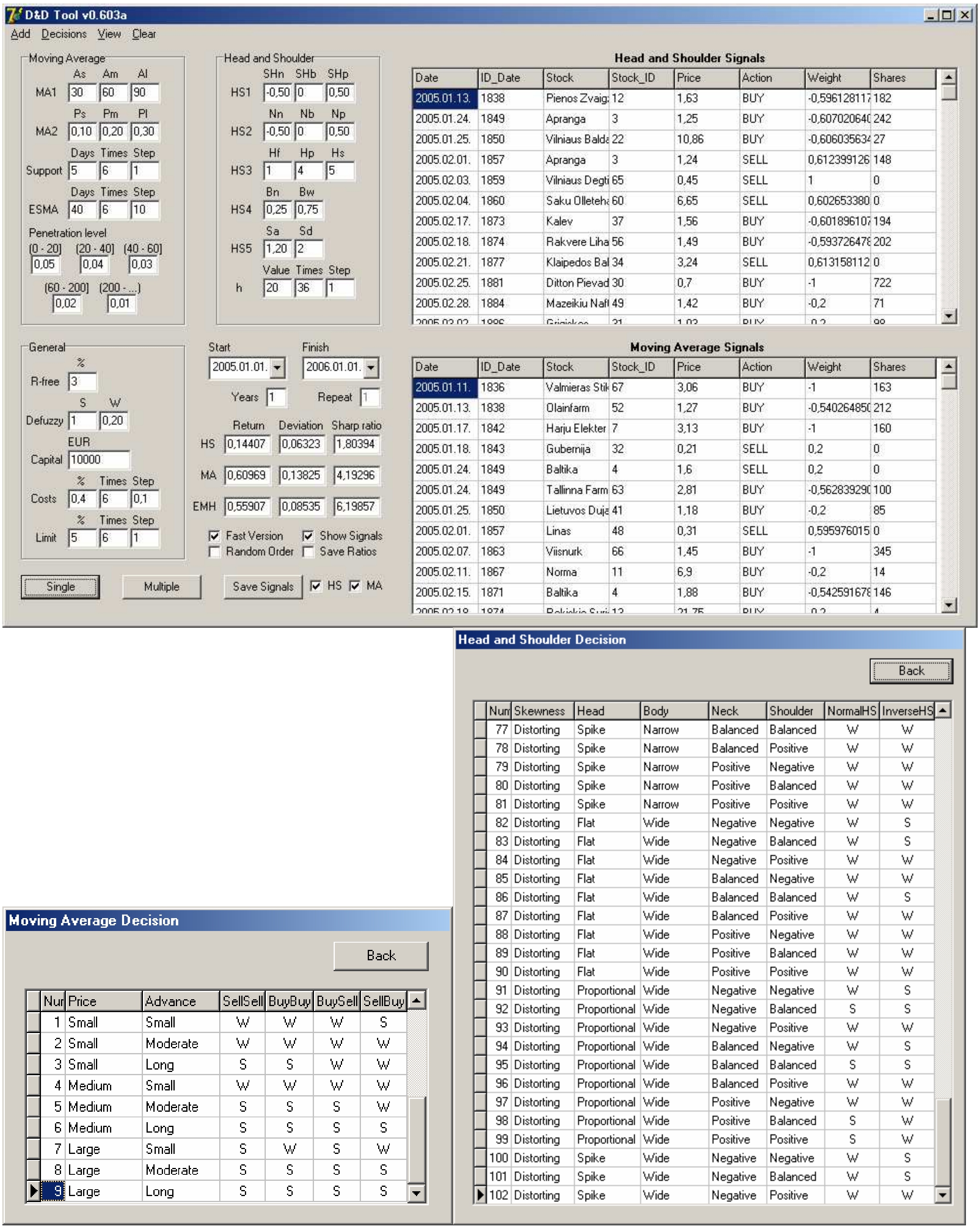

Fig. 6. Snapshots of the D&D Tool v0603a
Source: Created by authors

Appendix E. Program algorithm

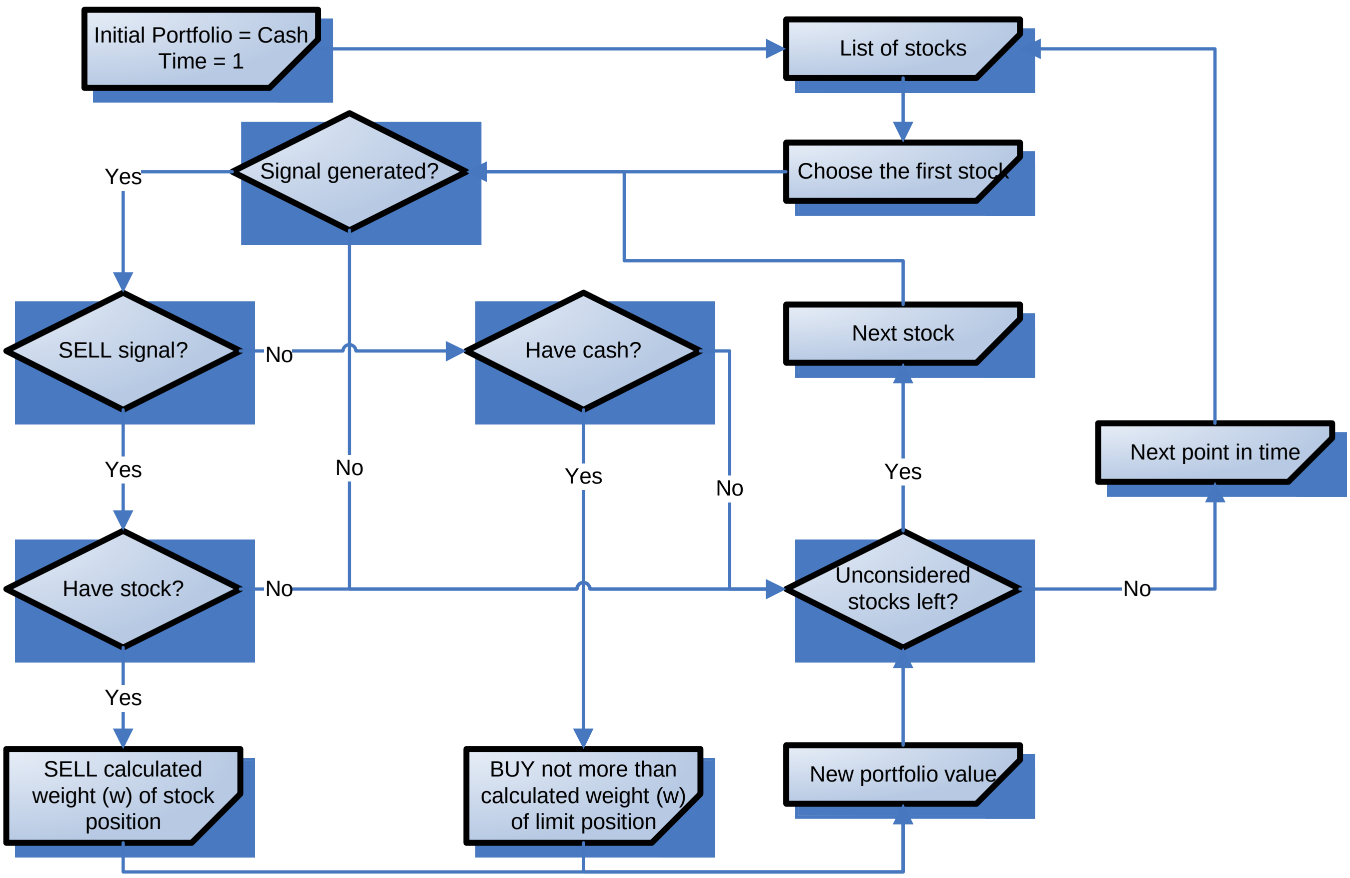

Fig. 7. Result computation algorithm
Source: Created by authors

Appendix F. List of stocks used in trading

Table 4. List of stocks

N	Name	Ticker	Listing	Country
1	Grindeks	GDR1R	Official	Latvia
2	Latvijas Gāze	GZE1R	Official	Latvia
3	Apranga	APG1L	Official	Lithuania
4	Baltika	BLT1T	Official	Estonia
5	Ekranas	EKR1L	Official	Lithuania
6	Eesti Telekom	ETLAT	Official	Estonia
7	Harju Elekter	HAE1T	Official	Estonia
8	Latvijas Kuģniecība	LSC1R	Official	Latvia
9	Lietuvos Telekomas	LTK1L	Official	Lithuania
10	Merko Ehitus	MKO1T	Official	Estonia
11	Norma	NRM1T	Official	Estonia
12	Pieno žvaigždės	PZV1L	Official	Lithuania
13	Rokiškio sūris	RSU1L	Official	Lithuania
14	SAF Tehnika	SAF1R	Official	Latvia
15	Sanitas	SAN1L	Official	Lithuania
16	Starman	SMN1T	Official	Estonia
17	Snaigė	SNG1L	Official	Lithuania
18	Tallink Group	TAL1T	Official	Estonia
19	Tallinna Kaubamaja	TKM1T	Official	Estonia
20	Tallinna Vesi	TVEAT	Official	Estonia
21	Utenos Trikotažas	UTR1L	Official	Lithuania
22	Vilniaus Baldai	VBL1L	Official	Lithuania
23	Venstpils Nafta	VNF1R	Official	Latvia
24	Vilniaus Vingis	VNG1L	Official	Lithuania
25	Alita	ALT1L	Second	Lithuania
26	Anykščių vynas	ANK1L	Second	Lithuania
27	Alytas Tekstilė	ATK1L	Second	Lithuania
28	Latvijas Balzams	BAL1R	Second	Latvia
29	Dvarčionių keramika	DKR1L	Second	Lithuania
30	Ditton pievadķēžu rūpnīca	DPK1R	Second	Latvia
31	Grigiškės	GRG1L	Second	Lithuania
32	Gubernija	GUB1L	Second	Lithuania
33	Invalda	IVL1L	Second	Lithuania
34	Klaipėdos Baldai	KBL1L	Second	Lithuania
35	Klaipėdos jūrų krovinių kompanija	KJK1L	Second	Lithuania
36	Klementi	KLEAT	Second	Estonia
37	Kalev	KLV1T	Second	Estonia
38	Kauno Energija	KNR1L	Second	Lithuania
39	Kauno Tiekimas	KTK1L	Second	Lithuania
40	Lisco Baltic Services	LBS1L	Second	Lithuania
41	Lietuvos Dujas	LDJ1L	Second	Lithuania
42	Lietuvos Elektrinė	LEL1L	Second	Lithuania
43	Lietuvos Energija	LEN1L	Second	Lithuania

44	Lifosa	LFO1L	Second	Lithuania
45	Lietuvos jūrų laivininkystė	LJL1L	Second	Lithuania
46	Lietuvos jūrų laivininkystė	LLK1L	Second	Lithuania
47	Liepājas metalurgs	LME1R	Second	Latvia
48	Linas	LNS1L	Second	Lithuania
49	Mažeikių nafta	MNF1L	Second	Lithuania
50	Mažeikių elektrinė	MZE1L	Second	Lithuania
51	Nord/LB Lietuva	NDL1L	Second	Lithuania
52	Olainfarm	OLF1R	Second	Latvia
53	Pramprojektas	PRM1L	Second	Lithuania
54	Panevėžio statybos trestas	PTR1L	Second	Lithuania
55	Rīgas kuģu būvētava	RKB1R	Second	Latvia
56	Rakvere Lihakombinaat	RLK1T	Second	Estonia
57	Rytų skirstomieji tinklai	RST1L	Second	Lithuania
58	Rīgas Transporta flote	RTF1R	Second	Latvia
59	Šiaulių bankas	SAB1L	Second	Lithuania
60	Saku Õlletehas	SKU1T	Second	Estonia
61	Snoras	SRS1L	Second	Lithuania
62	Stumbras	STU1L	Second	Lithuania
63	Tallinna Farmaatsiatehas	TFA1T	Second	Estonia
64	Ūkio bankas	UKB1L	Second	Lithuania
65	Vilniaus degtinė	VDG1L	Second	Estonia
66	Viisnurk	VNU1T	Second	Estonia
67	Valmieras Stikla Šķiedra	VSS1R	Second	Latvia
68	VST	VST1L	Second	Lithuania
69	Žemaitijos Pienas	ZMP1L	Second	Lithuania

Source: Created by authors

Appendix G. Model Parameters

Table 5. Summary of parameters

Group	Parameter	Parameter Description	Fuzzy Attributes of Parameter	Description of Fuzzy Attributes
MA	p	Filter. By how much percent the stock should be above/below moving average		
	n	Span. How many days we take for the base of the moving average		
	q	Filter. How many days the stock should be above/below moving average		
	MA1	Advance. A measure of how long a trend lasted before reversing.	A_S, A_M, A_L	Parameter may be small, moderate, and long. The values represent the boundaries.
	MA2	Price. How well the price performed during the trend before the trend reversed.	P_S, P_M, P_L	Parameter may be small, medium, and large. The values represent the boundaries.
HS	h	Smoothening. A sensitivity parameter for the smoothening.		
	HS1	Shoulder Line. Level of shoulder skew and direction of the line.	Sh_N, Sh_B, Sh_P	Parameter may be negative, balanced, and positive. The values represent the boundaries.
	HS2	Neck Line. Trough skew and direction of neckline.	N_N, N_B, N_P	Parameter may be negative, balanced, and positive. The values represent the boundaries.
	HS3	Head. How well the head is represented.	H_F, H_P, H_S	Parameter may be flat, proportional, or spiky. The values represent the boundaries.
	HS4	Body. How well the body of the pattern is represented.	$B_{N,}$ B_W	Parameter may be narrow or wide. The values represent the boundaries.
	HS5	Skew. How strongly is the pattern skewed along the a vertical mid-line	$S_{A,}$ S_D	Parameter may be acceptable or distorting. The values represent the boundaries.
Other	tc	Transaction Costs. Calculated as percentage of the value of transaction.		
	rf	Risk free rate which was applied to the funds held in cash		
	l	Position limit, i.e. how much could be invest as a percentage of current portfolio value		
			WT , ST	Boundary value for output functions, which allows getting back to crisp output from fuzzy numbers

Source: Created by authors

Appendix H. Results

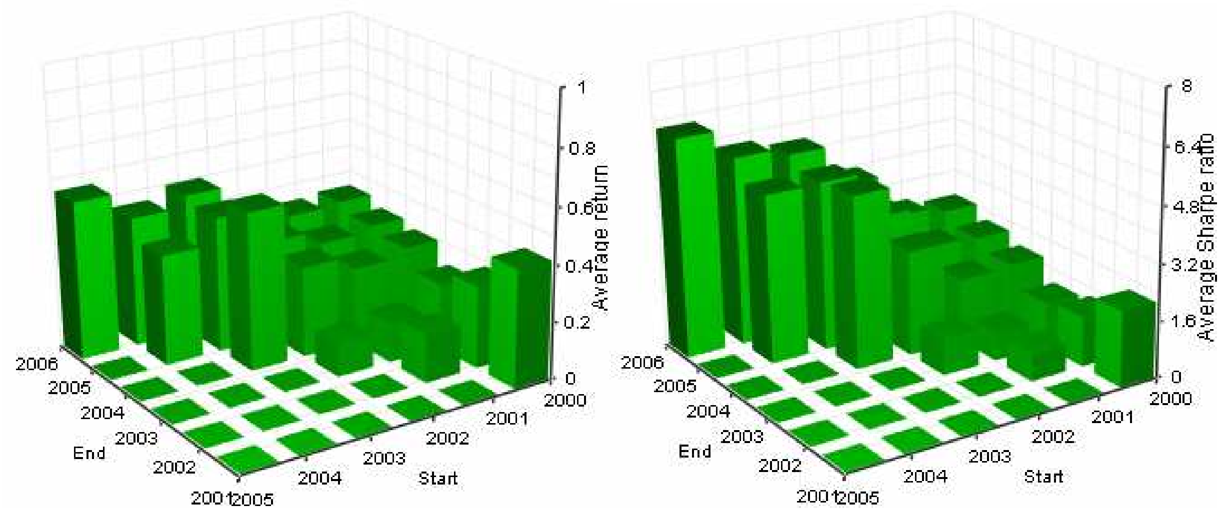

Fig. 8. Market returns
Source: Created by authors

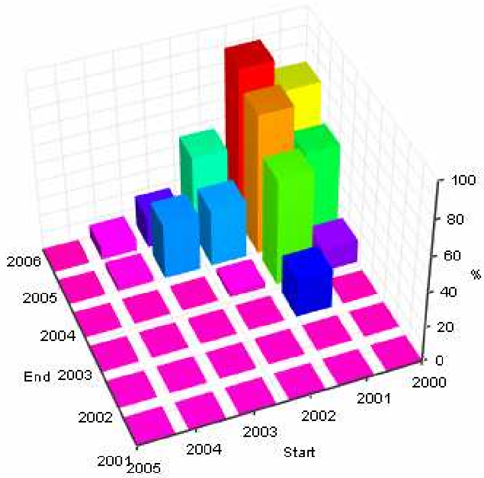

Fig. 9. Performance of the HS pattern over various intervals
Source: Created by authors

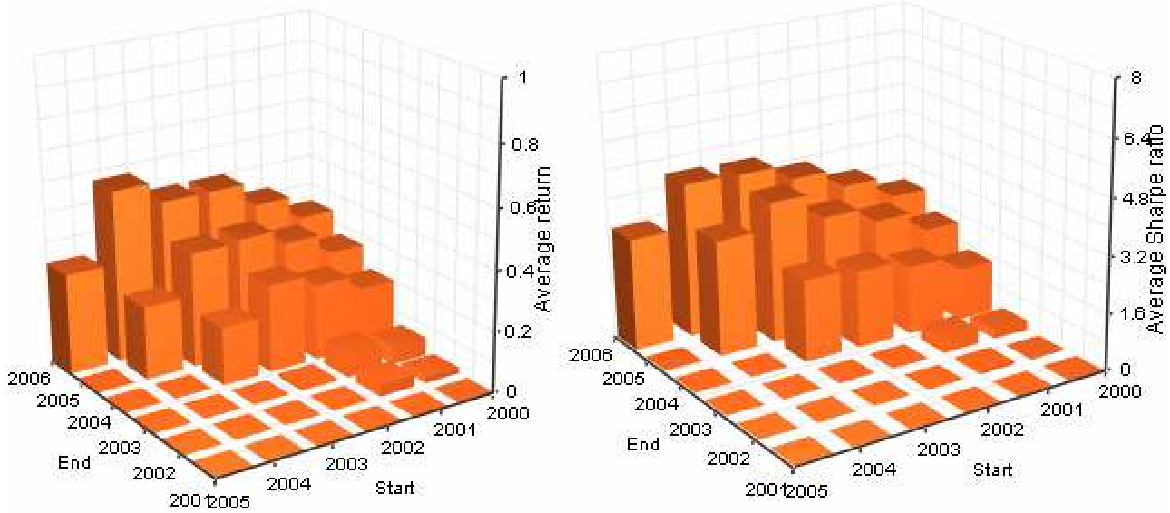

Fig. 9. Continuation
Source: Created by authors

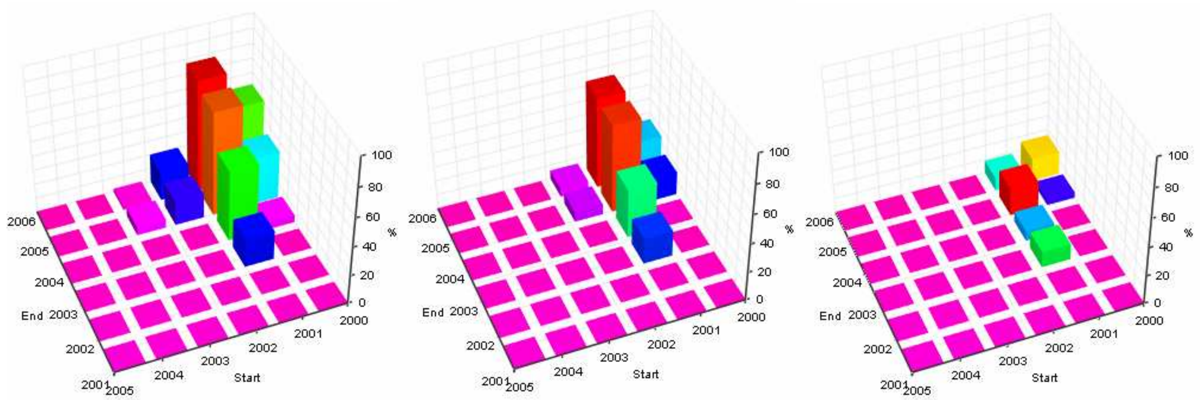

Fig. 10. Performance of the HS pattern at different margins
From left to right: HS outperformed the EMH by 10%, 20%, and 50%, respectively
Source: Created by authors

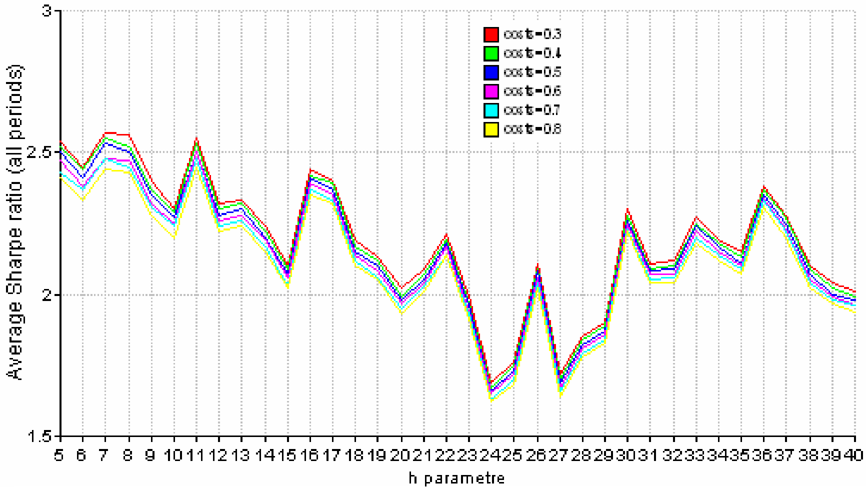

Fig. 11. Effect of parameter *h* on performance of the HS pattern
Source: Created by authors

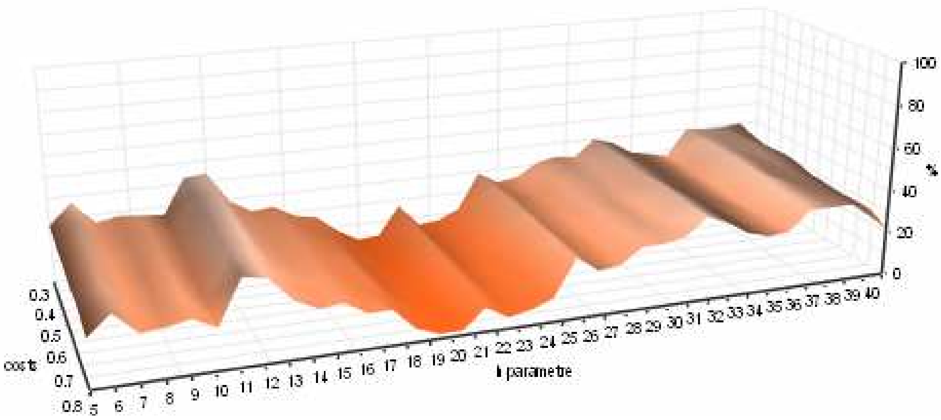

Fig. 12. Effects of *l* on HS performance
Source: Created by authors

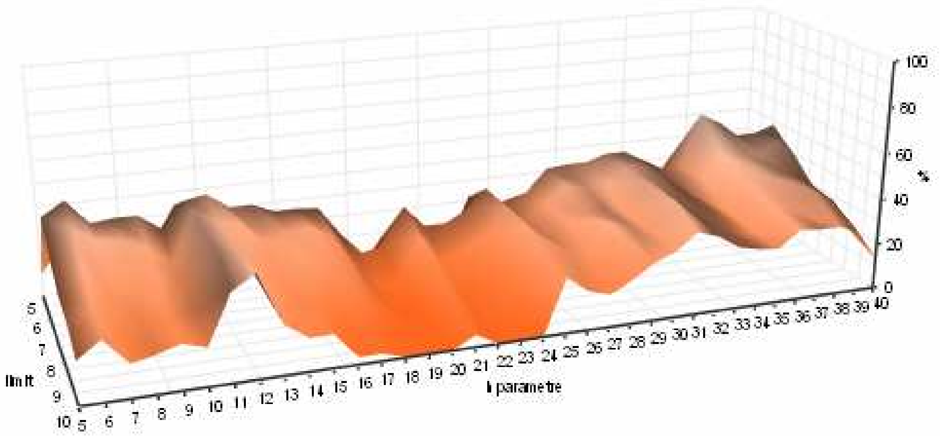

Fig. 13. Effects of *tc* on HS performance
Source: Created by authors

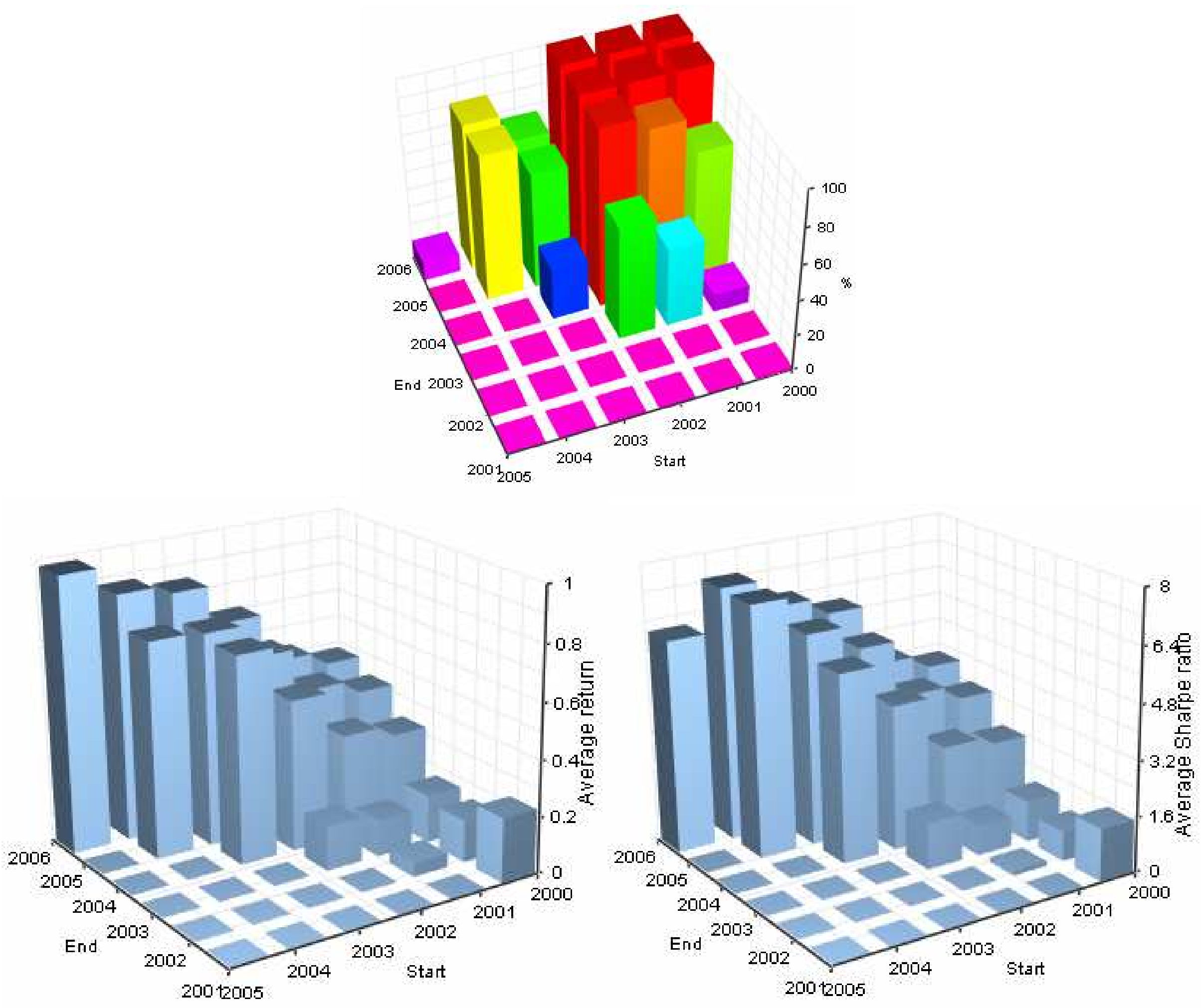

Fig. 14. Performance of the MA pattern over various intervals
Source: Created by authors

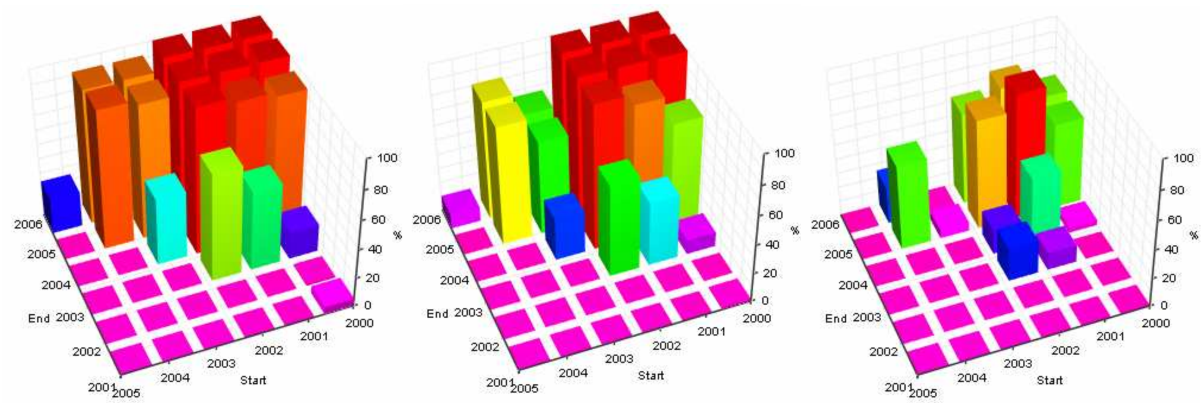

Fig. 15. Performance of the MA pattern at different margins
From left to right: MA outperformed the EMH by 10%, 20%, and 50%, respectively
Source: Created by authors

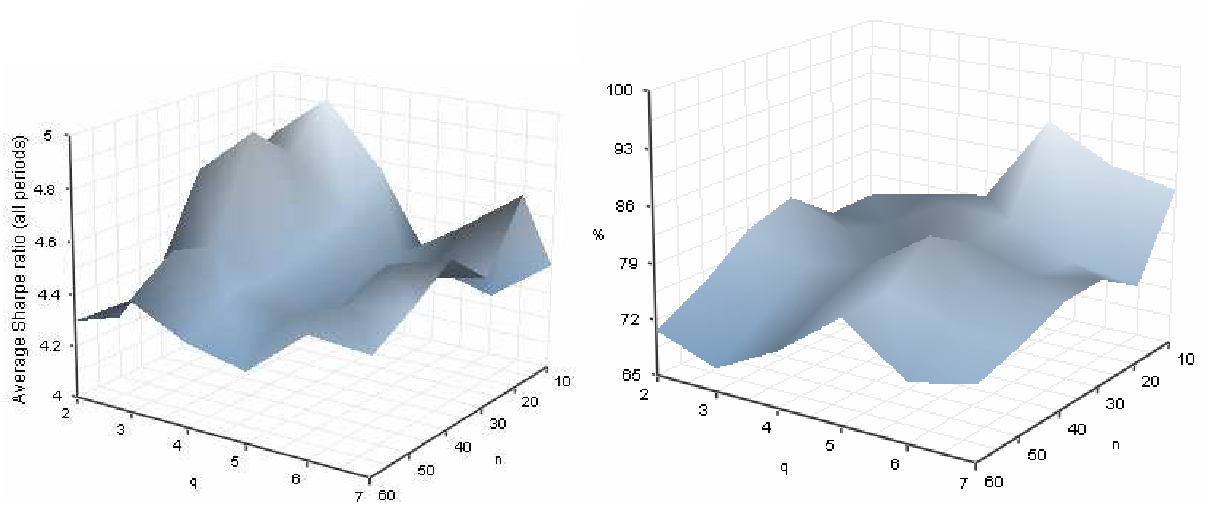

Fig. 16. Performance of the MA with respect to the parameters *n* and *q*
Source: Created by authors

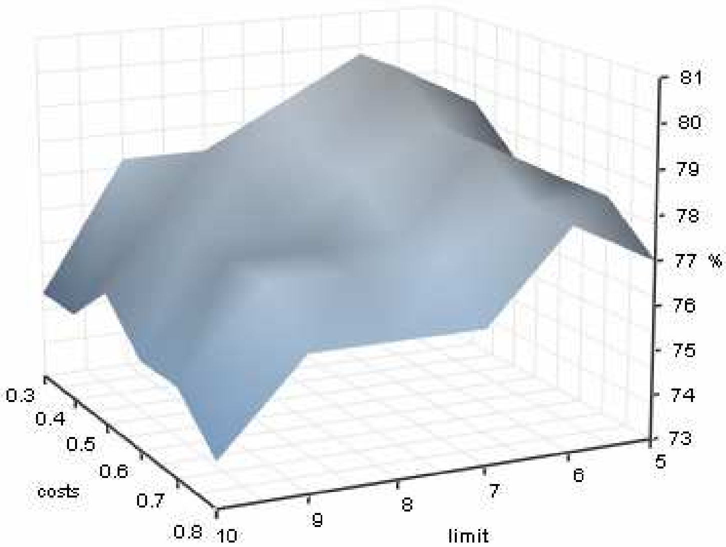

Fig. 17. Effect of *l* and *tc* on MA performance
Source: Created by authors

Appendix I. Test results

$$H_0 : \mu_{MA} - \mu_{EMH} = 0;$$
$$H_1 : \mu_{MA} - \mu_{EMH} > 0;$$
$$H_2 : \mu_{MA} - \mu_{EMH} < 0;$$
$$t - statistics = \frac{\mu_{MA} - \mu_{EMH}}{SE(\mu_{MA} - \mu_{EMH})}.$$

Table 6. Results of the MA Significance Tests
In bold: periods during which the MA which are significantly outperformed the EMH.

MA Sharp ratio								
Period	Observations	Mean	Alfa	t	critical value	H0	H1	H2
2000-2001	3888	-0,19	0,005	-150	2,58	Reject	Reject	Accept
2000-2002	3888	-0,14	0,005	-170	2,58	Reject	Reject	Accept
2000-2003	3888	-0,05	0,005	-100	2,58	Reject	Reject	Accept
2000-2004	3888	0,03	0,005	56,45	2,58	Reject	Accept	Reject
2000-2005	3888	0,11	0,005	221,65	2,58	Reject	Accept	Reject
2000-2006	3888	0,13	0,005	252,65	2,58	Reject	Accept	Reject
2001-2002	3888	-0,12	0,005	-140	2,58	Reject	Reject	Accept
2001-2003	3888	-0,01	0,005	-20,63	2,58	Reject	Reject	Accept
2001-2004	3888	0,01	0,005	176,08	2,58	Reject	Accept	Reject
2001-2005	3888	0,18	0,005	368,05	2,58	Reject	Accept	Reject
2001-2006	3888	0,19	0,005	455,73	2,58	Reject	Accept	Reject
2002-2003	3888	0,04	0,005	62,61	2,58	Reject	Accept	Reject
2002-2004	3888	0,22	0,005	344,10	2,58	Reject	Accept	Reject
2002-2005	3888	0,29	0,005	576,02	2,58	Reject	Accept	Reject
2002-2006	3888	0,32	0,005	644,08	2,58	Reject	Accept	Reject
2003-2004	3888	0,18	0,005	125,50	2,58	Reject	Accept	Reject
2003-2005	3888	0,28	0,005	247,42	2,58	Reject	Accept	Reject
2003-2006	3888	0,35	0,005	376,50	2,58	Reject	Accept	Reject
2004-2005	3888	0,38	0,005	141,06	2,58	Reject	Accept	Reject
2004-2006	3888	0,42	0,005	242,89	2,58	Reject	Accept	Reject
2005-2006	3888	0,42	0,005	125,18	2,58	Reject	Accept	Reject
All	81648	0,15	0,005	215,66	2,58	Reject	Accept	Reject

Table 6. Continuation

MA Return								
Period	Observations	Mean	Alfa	t	critical value	H0	H1	H2
2000-2001	3888	-0,19	0,005	-150	2,58	Reject	Reject	Accept
2000-2002	3888	-0,14	0,005	-170	2,58	Reject	Reject	Accept
2000-2003	3888	-0,05	0,005	-100	2,58	Reject	Reject	Accept
2000-2004	3888	0,03	0,005	56,45	2,58	Reject	Accept	Reject
2000-2005	3888	0,11	0,005	221,65	2,58	Reject	Accept	Reject
2000-2006	3888	0,13	0,005	252,65	2,58	Reject	Accept	Reject
2001-2002	3888	-0,12	0,005	-140	2,58	Reject	Reject	Accept
2001-2003	3888	-0,01	0,005	-20,63	2,58	Reject	Reject	Accept
2001-2004	3888	0,01	0,005	176,08	2,58	Reject	Accept	Reject
2001-2005	3888	0,18	0,005	368,05	2,58	Reject	Accept	Reject
2001-2006	3888	0,19	0,005	455,73	2,58	Reject	Accept	Reject
2002-2003	3888	0,04	0,005	62,61	2,58	Reject	Accept	Reject
2002-2004	3888	0,22	0,005	344,10	2,58	Reject	Accept	Reject
2002-2005	3888	0,29	0,005	576,02	2,58	Reject	Accept	Reject
2002-2006	3888	0,32	0,005	644,09	2,58	Reject	Accept	Reject
2003-2004	3888	0,18	0,005	125,50	2,58	Reject	Accept	Reject
2003-2005	3888	0,28	0,005	247,42	2,58	Reject	Accept	Reject
2003-2006	3888	0,35	0,005	376,50	2,58	Reject	Accept	Reject
2004-2005	3888	0,38	0,005	141,06	2,58	Reject	Accept	Reject
2004-2006	3888	0,42	0,005	242,89	2,58	Reject	Accept	Reject
2005-2006	3888	0,42	0,005	125,18	2,58	Reject	Accept	Reject
All	81648	0,15	0,005	215,66	2,58	Reject	Accept	Reject

Source: Created by authors

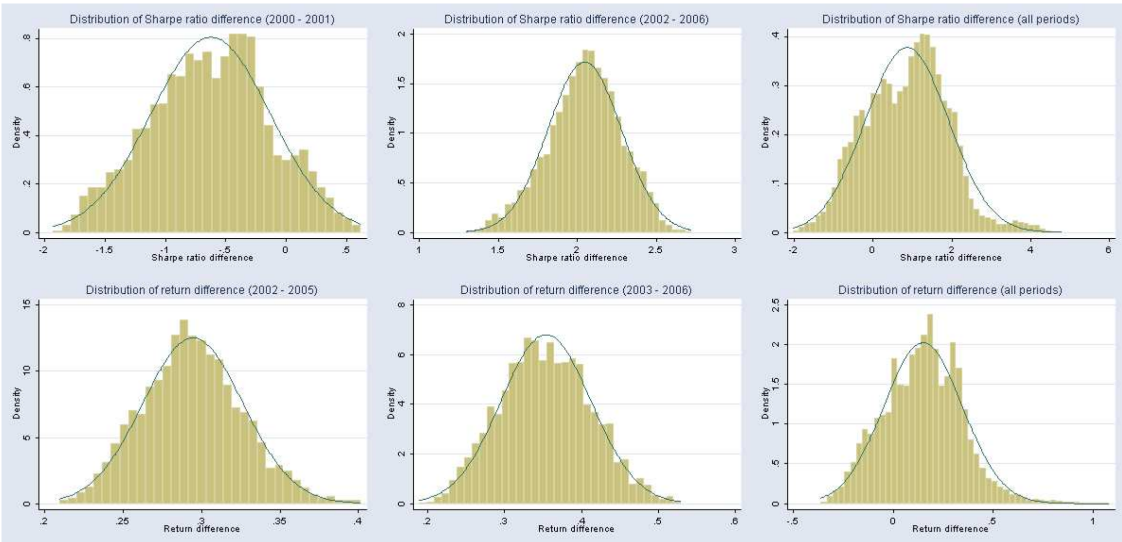

Fig. 18. Examples of empirical distribution of the performance differences for the MA pattern
Source: Created by authors

$$H_0 : \mu_{HS} - \mu_{EMH} = 0;$$
$$H_1 : \mu_{HS} - \mu_{EMH} > 0;$$
$$H_2 : \mu_{HS} - \mu_{EMH} < 0;$$
$$t - statistics = \frac{\mu_{HS} - \mu_{EMH}}{SE(\mu_{HS} - \mu_{EMH})}.$$

Table 7. Results of the HS Significance Tests
In bold: periods during which the HS significantly outperformed the EMH.

HS Sharp ratio								
Period	Observations	Mean	Alfa	t	critical value	H0	H1	H2
2000-2001	3888	-2,65	0,005	-270	2,58	Reject	Reject	Accept
2000-2002	3888	-1,52	0,005	-290	2,58	Reject	Reject	Accept
2000-2003	3888	-0,10	0,005	-220	2,58	Reject	Reject	Accept
2000-2004	3888	-0,37	0,005	-71,36	2,58	Reject	Reject	Accept
2000-2005	3888	0,01	0,025	2,03	1,96	Reject	Accept	Reject
2000-2006	3888	0,22	0,005	33,45	2,58	Reject	Accept	Reject
2001-2002	3888	-0,70	0,005	-120	2,58	Reject	Reject	Accept
2001-2003	3888	-0,35	0,005	-45,46	2,58	Reject	Reject	Accept
2001-2004	3888	0,14	0,005	17,26	2,58	Reject	Accept	Reject
2001-2005	3888	0,55	0,005	59,28	2,58	Reject	Accept	Reject
2001-2006	3888	0,67	0,005	82,08	2,58	Reject	Accept	Reject
2002-2003	3888	-1,33	0,005	-220	2,58	Reject	Reject	Accept
2002-2004	3888	-0,79	0,005	-110	2,58	Reject	Reject	Accept
2002-2005	3888	-0,24	0,005	-24,23	2,58	Reject	Reject	Accept
2002-2006	3888	-0,08	0,005	-9,20	2,58	Reject	Reject	Accept
2003-2004	3888	-2,58	0,005	-530	2,58	Reject	Reject	Accept
2003-2005	3888	-0,81	0,005	-55,50	2,58	Reject	Reject	Accept
2003-2006	3888	-0,81	0,005	-67,95	2,58	Reject	Reject	Accept
2004-2005	3888	-1,50	0,005	-110	2,58	Reject	Reject	Accept
2004-2006	3888	-0,98	0,005	-85,32	2,58	Reject	Reject	Accept
2005-2006	3888	-3,03	0,005	-200	2,58	Reject	Reject	Accept
All	81648	-0,82	0,005	-200	2,58	Reject	Reject	Accept

Table 7. Continuation

HS return								
Period	Observations	Mean	Alfa	t	critical value	H0	H1	H2
2000-2001	3888	-0,43	0,005	-540	2,58	Reject	Reject	Accept
2000-2002	3888	-0,27	0,005	-490	2,58	Reject	Reject	Accept
2000-2003	3888	-0,17	0,005	-370	2,58	Reject	Reject	Accept
2000-2004	3888	-0,10	0,005	-210	2,58	Reject	Reject	Accept
2000-2005	3888	-0,05	0,005	-80,43	2,58	Reject	Reject	Accept
2000-2006	3888	-0,01	0,005	-14,81	2,58	Reject	Reject	Accept
2001-2002	3888	-0,14	0,005	-240	2,58	Reject	Reject	Accept
2001-2003	3888	-0,07	0,005	-88,21	2,58	Reject	Reject	Accept
2001-2004	3888	-0,02	0,005	-20,31	2,58	Reject	Reject	Accept
2001-2005	3888	0,04	0,005	38,41	2,58	Reject	Accept	Reject
2001-2006	3888	0,08	0,005	88,09	2,58	Reject	Accept	Reject
2002-2003	3888	-0,11	0,005	-180	2,58	Reject	Reject	Accept
2002-2004	3888	-0,04	0,005	-39,64	2,58	Reject	Reject	Accept
2002-2005	3888	0,04	0,005	36,60	2,58	Reject	Accept	Reject
2002-2006	3888	0,11	0,005	107,23	2,58	Reject	Accept	Reject
2003-2004	3888	-0,37	0,005	-390	2,58	Reject	Reject	Accept
2003-2005	3888	-0,10	0,005	-46,62	2,58	Reject	Reject	Accept
2003-2006	3888	0,004	0,025	2,20	1,96	Reject	Accept	Reject
2004-2005	3888	-0,15	0,005	-110	2,58	Reject	Reject	Accept
2004-2006	3888	0,11	0,005	47,01	2,58	Reject	Accept	Reject
2005-2006	3888	-0,23	0,005	-130	2,58	Reject	Reject	Accept
All	81648	-0,09	0,005	-160	2,58	Reject	Reject	Accept

Source: Created by authors

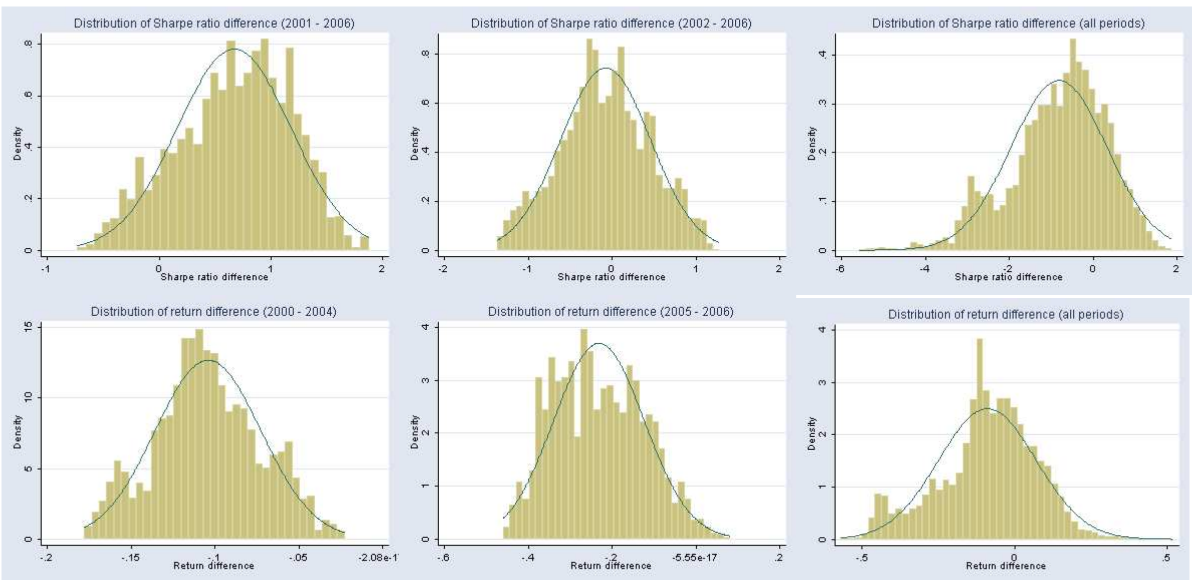

Fig. 19. Examples of empirical distribution of the performance differences for the HS pattern
Source: Created by authors