Maths Foundation
Seminar 1 - Solution
Stockholm School of Economics in Riga, February 2022

Exercise 1

1. (a) Positive
   (b) Negative
   (c) 0
2. (a) 0
   (b) Positive
   (c) Negative
3. (a) Positive
   (b) Negative
   (c) 0
4. (a) Negative
   (b) 0
   (c) Negative

Exercise 2

Find the average rate of change of the following functions between the given pairs of x-values.

($\Leftrightarrow$ slope of the line passing by $f(x_1)$ and $f(x_2)$?)

1. $f(x) = x^2 + x$: 5; 4; 3,1; 3,01.
2. $f(x) = 2x^2 + x - 2$: 13; 11; 9,2; 9,02.

Exercise 3

1. Because of the division by 0.
2. Find the instantaneous rate of change of the following functions at the given x-values, computed as the limit of the Newton quotient (see the exercise above). Compare your answers with what you obtained in exercise 2.

   (a) $f'(1) = 3$
   (b) $f'(2) = 9$

Exercise 4

1. The population is decreasing.
2. The temperature has been increasing over the first day, and decreased during the second day.
3. It is at this moment that the temperature is the lowest.

Exercise 5

1. $f'(x) = 5x^4$
2. $f'(x) = 500x^{499}$
3. $f'(x) = \frac{1}{2}x^{-1/2}$
4. $f'(x) = 2x^{-2/3}$
5. $f'(x) = 12x^2 - 2$
6. $f'(x) = x^{-2/3}$, so $f'(x) = -\frac{2}{3}x^{-5/3}$
7. $f'(x) = \frac{1}{2}x^2 + x + 1$.

Exercise 6

1. The slope of the tangent is given by the derivative of $f(x)$ at $x = 3$. The derivative of $f(x)$ is $f'(x) = 2x - 2$, so $f'(3) = 4$. So the equation of the tangent is $y = 4x + b$. Also, we know that the tangent passes through $(3, f(3))$, and $f(3) = 5$. So $5 = 4 \times 3 + b \Rightarrow b = -7$.

   Note: to go faster, you can remember that the equation of a tangent to the graph of $y = f(x)$ at the point $(a, f(a))$ is $y - f(a) = f'(a)(x - a)$.
2. $f'(x) = 3x^2 - 6x + 2$, so $f'(2) = 2$, and $f(2) = -2$. Applying the aforementioned formula, we obtain: $y = 2x - 6$.

Exercise 7

1. The derivative of $f(x)$ is $f'(x) = -x^2 + 50x - 300$. The rate of change in 2016 is given by $f'(0) = -300$: in 2016, the population decreases by 300000 inhabitants.
2. This is given $f'(20) = -20^2 + 50.20 - 300 = 300$. In 2036, the population is expected to increase by 300000 inhabitants.

Exercise 8

1. It represent the "satisfaction" associated with the possession of x euros.
2. $MU(x) = 50x^{-1/2}$. This represents the increase in satisfaction for each additional euro you get.
3. $MU(1) = 50$, and $MU(1000000) = 0,5$. Marginal utility of money is decreasing.

Exercise 9

1. We simply have to develop:
$(\sqrt{x+h}-\sqrt{x})(\sqrt{x+h}+\sqrt{x}) = x+h+\sqrt{x+h}\sqrt{x}-\sqrt{x+h}\sqrt{x}-x = h.$
2. Using the result just above, we can substitute h:
$\frac{\sqrt{x+h}-\sqrt{x}}{(\sqrt{x+h}+\sqrt{x})(\sqrt{x+h}-\sqrt{x})} = \frac{1}{\sqrt{x+h}+\sqrt{x}}.$
3. Recall that the definition of a derivative is: $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$.
Using the previous results, we can rewrite:
$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{1}{\sqrt{x+h}+\sqrt{x}} = \frac{1}{2\sqrt{x}}.$