Seminar 2 : introduction to optimization

Exercise 1

For each function, find $f'(x)$, $f''(x)$ and $f'''(x)$.

1. $f(x) = x^{1/3}$
2. $f(x) = x^4 - 3x^3 + 2x^2 - 8x + 4$
3. $f(x) = \sqrt{x} - \frac{1}{x}$
4. $f(x) = -3x^2 - 5x + 4$

Exercise 2

Suppose that the quantity described in the following situations is represented by a function $f(t)$ where t stands for time. Is the first derivative positive or negative ? Is the second derivative positive or negative ? Represent each situation in a graph.

1. The economy is growing, but more slowly.
2. The temperature is dropping increasingly rapidly.
3. The stock marker is declining, but less rapidly.
4. The population is growing increasingly fast.

Exercise 3

The national debt of a country t years from now is predicted to be $D(t) = 65 + 9t^{4/3}$ billion euros. Find $D'(8)$ and $D''(8)$ and interpret.

Exercise 4

1. What is a stationary point ?
2. How to classify a stationary point ?

Exercise 5

Consider the function $f(x) = x^3 - 3x^2 - 9x + 10$.

1. Find the stationary points.
2. Find where the function is increasing and where the function is decreasing.
3. Compute $f(0)$, $f(-1)$ and $f(3)$, and sketch the graph of the function.

Exercise 6

A plane is to take off and reach level of cruising altitude of 5 km after a horizontal distance of 100 km. Find a polynomial flight path of the form $f(x) = ax^3 + bx^2 + cx + d$, knowing that :

— The plane is on the ground at $x = 0$.
— The plane is horizontal at $x = 0$.
— At the 100^{th} km, the height is 5 and the path is horizontal.

Exercise 7

A country is hit by a flu epidemic. The number of ill persons on day t is $f(t) = 30t^2 - t^3$ (for $0 \leq t \leq 30$), where t is the number of days since the apparition of the virus.

1. What does $f'(t)$ represent ?
2. Compute $f'(10)$, and interpret it.
3. Study the variation of the number of ill persons.
4. When is the total number of ill persons maximum ?
5. When is the disease spreading at the highest pace ?