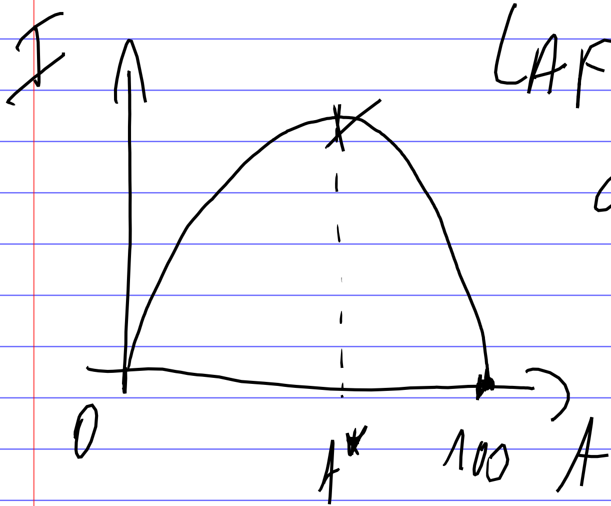
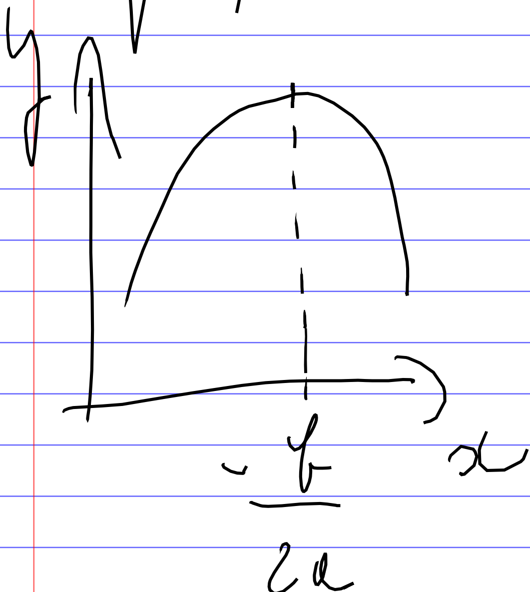
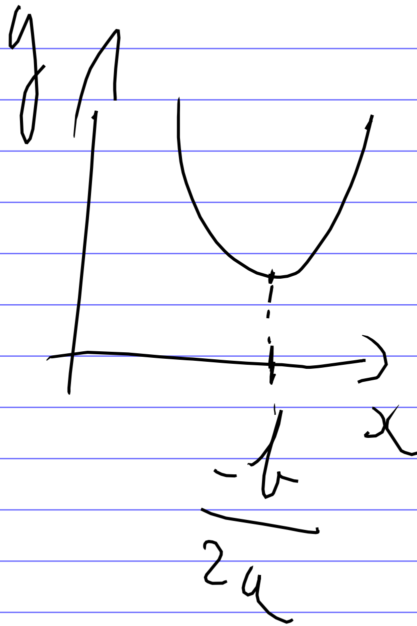
LAFFER CURVE



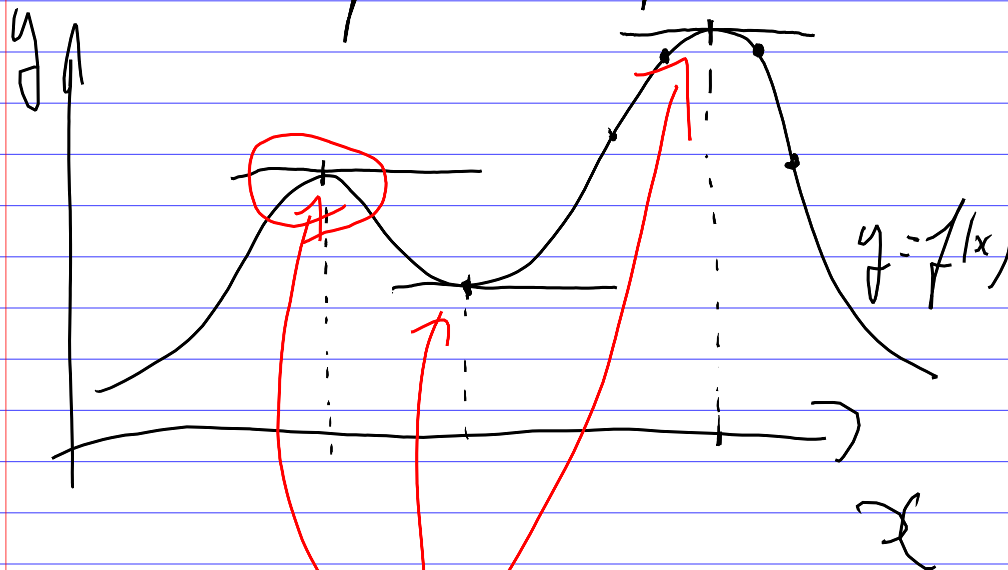
Quadratic function

$$f(x) = ax^2 + bx + c$$

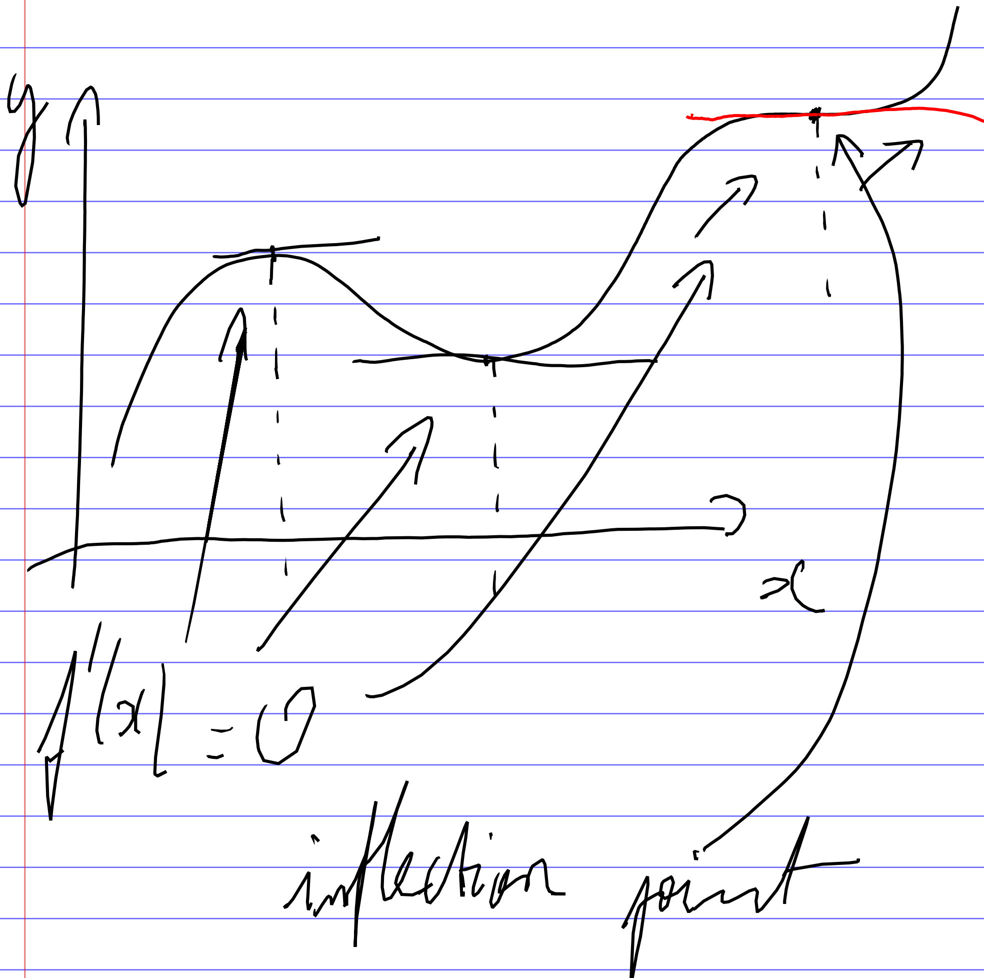


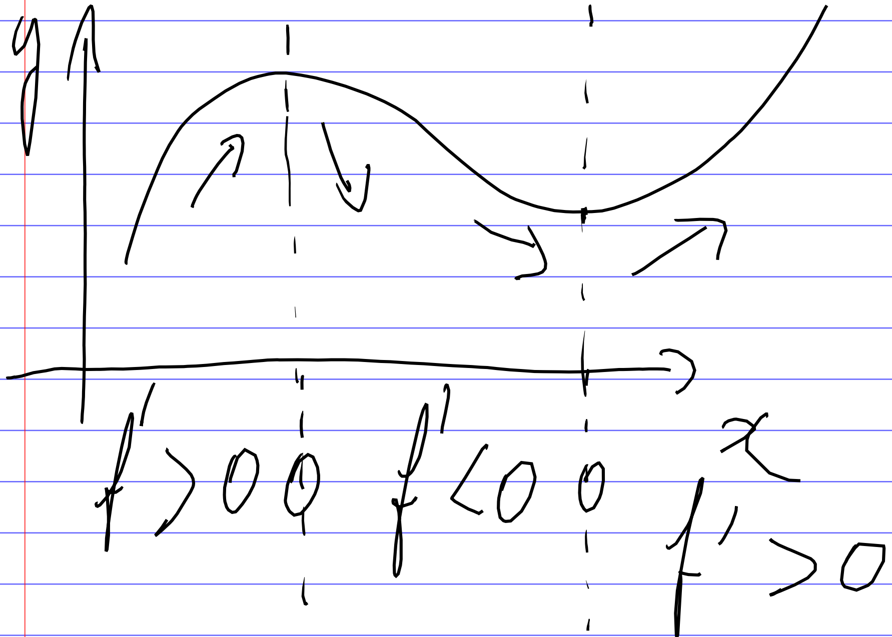


More complicated function?



$f'(x) = 0$


y
x
f'(x) = 0
inflection point


y
x
f' > 0
0
f' < 0
0
f' > 0

$$f(x) = x^3 - 12x + 8$$

$\rightarrow$ Stationary points?

$$f'(x) = 3x^2 - 12 = 0$$

$$3x^2 = 12$$

$$x^2 = 4$$

$$x = \pm 2$$

x		-2		2	
$f(x)$	$\nearrow$		$\searrow$		$\nearrow$
$f'(x)$	$+$	0	$-$	0	$+$

$$f'(0) = -12 < 0$$

$$f'(-3) > 0 \quad ; \quad f'(3) > 0$$

$\rightarrow$ So we have a maximum at $x = -2$ and a minimum at $x = 2$

Higher order derivatives

$f(x) = x^3 - 6x^2 + 2x + 7$

$f'(x) = 3x^2 - 12x + 2$

$f''(x) = 6x - 12$

$f'''(x) = 6$

$$P(A) = 6250 + 160A^{3/4}$$

1) $$P'(A) = \frac{3}{4} \cdot 160A^{-1/4}$$

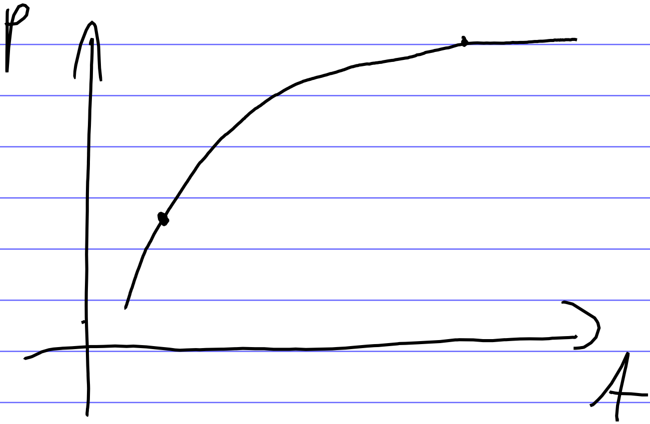
$$= 120A^{-1/4}$$

So $P'(16) = 60$

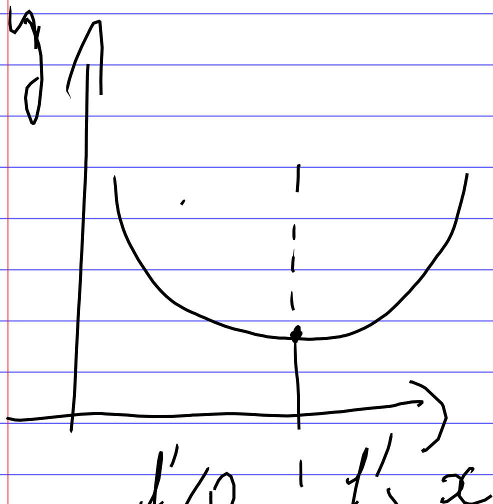
2) $$P''(A) = -\frac{1}{4} \cdot 120A^{-5/4}$$

$$-30A^{-5/4}$$

$P''(16) \simeq -0{,}94$



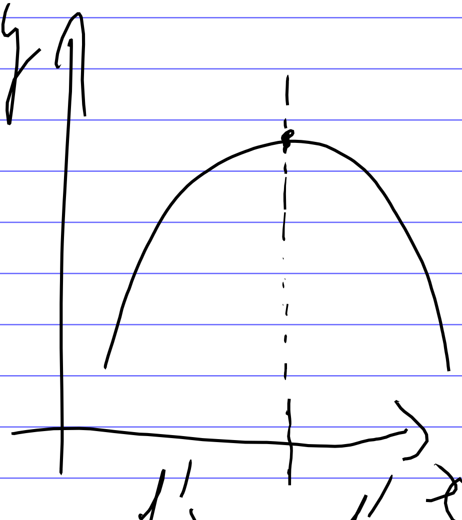
→ The population keeps increasing but slower and slower...



$f' < 0 \quad f' > 0$

$f'' > 0$

$\Rightarrow$ convexe



$f' > 0 \quad f' < 0$

$f'' < 0$

$\Rightarrow$ concave

$$f(x) = x^3 - 9x^2 + 15x$$

$$f'(x) = 3x^2 - 18x + 15 = 0$$

$$\Rightarrow f'(x) = 0$$

when $x = 1$ or $x = 5$

$$f''(x) = 6x - 18$$

$f''(1) = 6 \times 1 - 18 = -12 < 0$

↳ Maximum

$f''(5) = 6 \cdot 5 - 18 = 12 > 0$

↳ Minimum