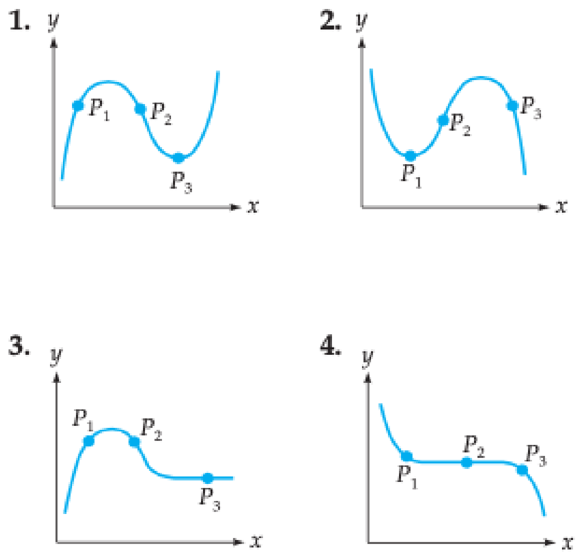
Seminar 1 : Introduction to differentiation

Exercise 1

By imagining tangent lines at points P_1, P_2 and P_3, state whether the slopes are positive, zero or negative at these points.



Exercise 2

Find the average rate of change of the following functions between the given pairs of x-values.

1. $f(x) = x^2 + x$
   (a) at $x = 1$ and $x = 3$
   (b) at $x = 1$ and $x = 2$
   (c) at $x = 1$ and $x = 1.1$
   (d) at $x = 1$ and $x = 1.01$
2. $f(x) = 2x^2 + x - 2$
   (a) at $x = 2$ and $x = 5$
   (b) at $x = 2$ and $x = 4$
   (c) at $x = 2$ and $x = 3.1$
   (d) at $x = 2$ and $x = 3.01$

Exercise 3

1. When we calculate the derivative using the formula

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h},$$

   why don't we simply substitute $h = 0$ into the formula ?

2. Find the instantaneous rate of change of the following functions at the given x-values, computed as the limit of the Newton quotient (see the exercise above). Compare your answers with what you obtained in exercise 2.
   (a) $f(x) = x^2 + x$ at $x = 1$
   (b) $f(x) = 2x^2 + x - 2$ at $x = 2$

Exercise 4

1. The population of a city in year x is given by a function whose derivative is negative. What does this mean about this city ?
2. A patient's temperature at time x hours is given by a function whose derivative is positive for $0 < x < 24$ and negative for $24 < x < 48$. Assuming that the patient begins and ends with a normal temperature, is the patient's health improving or deteriorating during the first day ? During the second day ?
3. Suppose that the temperature outside your house x hours after midnight is given by a function whose derivative is negative for $0 < x < 6$ and positive for $6 < x < 12$. What can you say about the temperature at time 6 a.m. compared to the rest of the day ?

Exercise 5

Find the derivative of each function.

1. $f(x) = x^5$
2. $f(x) = x^{500}$
3. $f(x) = x^{1/2} + 100000$ (hint : why must the derivative of a constant function (for example, $f(x) = 2$) be equal to 0 ?)
4. $f(x) = 6\sqrt[3]{x}$
5. $f(x) = 4x^3 - 2x + 10$
6. $f(x) = \frac{1}{x^{2/3}}$
7. $f(x) = \frac{1}{6}x^3 + \frac{1}{2}x^2 + x + 1$

Exercise 6

1. Find the equation of the tangent line to $f(x) = x^2 - 2x + 2$ at $x = 3$.
2. Find the equation of the tangent line to $f(x) = x^3 - 3x^2 + 2x - 2$ at $x = 2$.

Exercise 7

You are interested in forecasting the size of the population of a country. You estimated that the population is given by

$$P(x) = -\frac{1}{3}x^3 + 25x^2 - 300x + 31000$$

(in thousands), where x is the number of years after 2016.

1. What will be the rate of change of the population in 2016 ? Interpret.
2. What will the rate of change of the population in 2036 ? Interpret.

Exercise 8

Generally, the more you have of something, the less valuable each additional unit becomes. For example, a dollar is less valuable to a millionaire than to a homeless person. Economists define a person's **utility function** $U(x)$ for a product as the "satisfaction" of having x units of this good. The *derivative* of $U(x)$ is called the *marginal utility* function, $MU(x) = U'(x)$. Suppose that a person's utility function for money is given by $100\sqrt{x}$.

1. In this example, what does $U(x)$ represent ?
2. Find the marginal utility function $MU(x)$. How can you interpret this function ?
3. Compute $MU(1)$ and $MU(1000000)$. How do they compare ?

Exercise 9 (harder)

1. Show that $(\sqrt{x+h} - \sqrt{x})(\sqrt{x+h} + \sqrt{x}) = h$.
2. If $f(x) = \sqrt{x}$, show that $(f(x+h) - f(x))/h = 1/(\sqrt{x+h} + \sqrt{x})$.
3. Use these results to show that for $x > 0$,
$$f(x) = \sqrt{x} \Rightarrow f'(x) = \tfrac{1}{2\sqrt{x}}.$$

Thanks for coming, see you next Saturday !
Could you please take 2 minutes to fill in this survey ?
http ://ejuz.lv/sseriga1