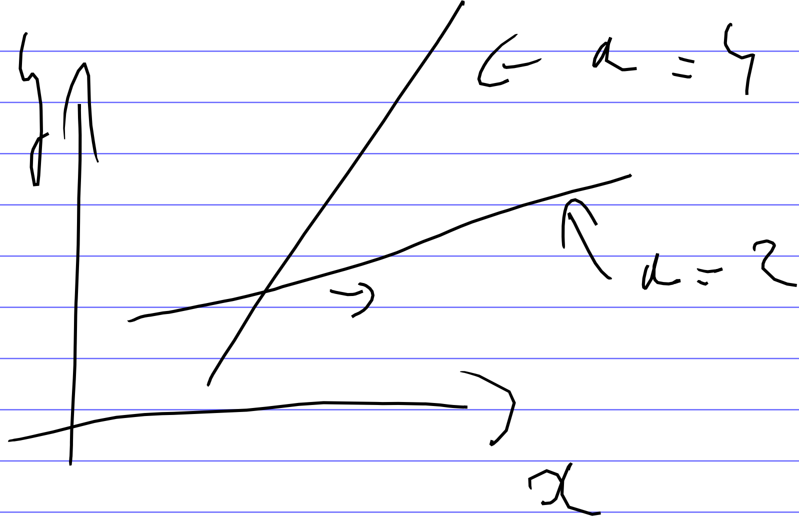
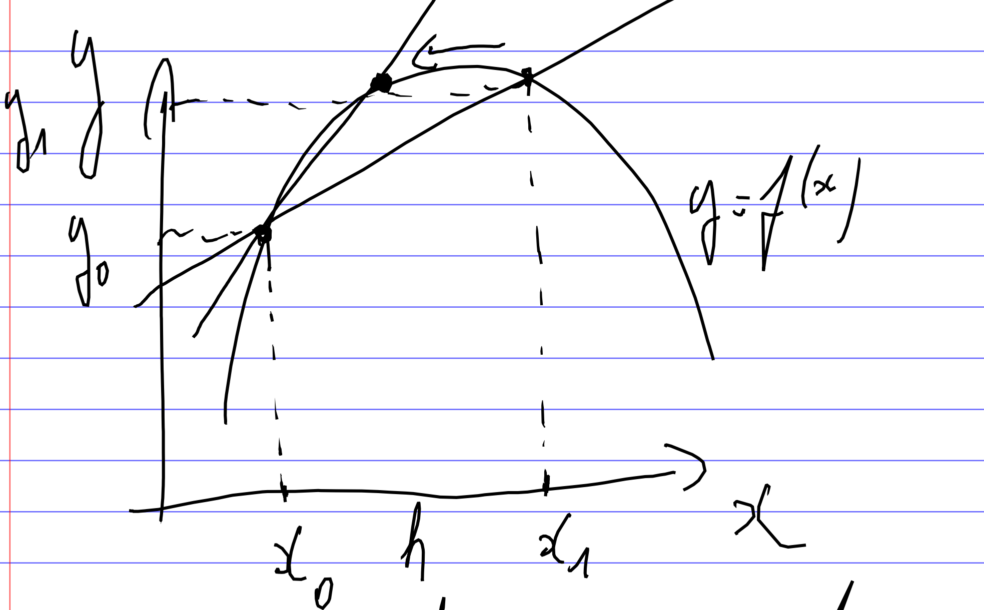
$\boxed{\begin{matrix} m \\ n \end{matrix}}$

$f(x) = y$

input
independent
variable

output
dependent
variable


y
α = 4
α = 2
x



What is the rate of change at x_0?

Compute $\dfrac{y_1 - y_0}{x_1 - x_0}$?

Let's denote the distance between x_0 and $x_1 \to h$

So $x_1 = x_0 + h$

So we can write:

$$\underbrace{\frac{y_1 - y_0}{x_1 - x_0} = \frac{f(x_0+h) - f(x_0)}{h}}$$

NEWTON

QUOTIENT

h infinitely small?

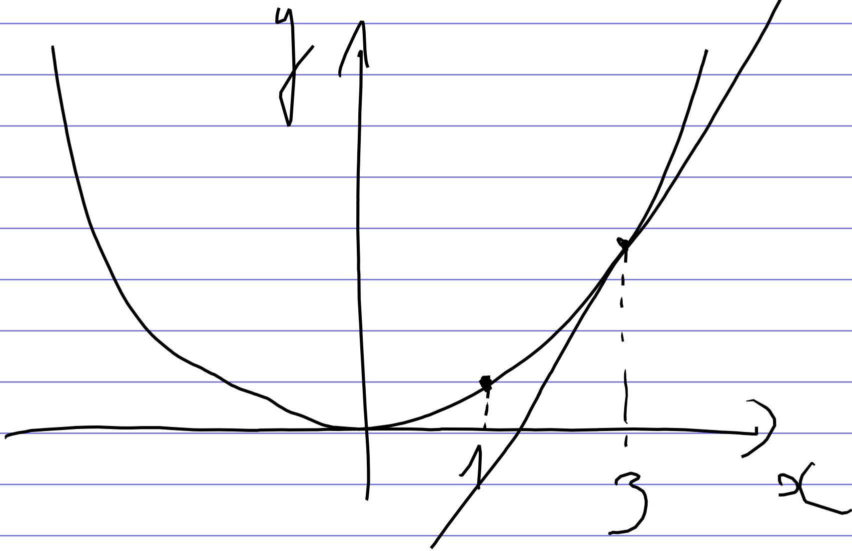
$$\lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h} = f'(x_0)$$

This gives us the slope of the tangent at x_0

$\Rightarrow$ The derivative

$f(x) = x^2$

Derivative at $x = 3$?



$$\frac{f(3+h)-f(3)}{h}$$

$$=\frac{(3+h)^2-3^2}{h}$$

$$=\frac{9+6h+h^2-9}{h}$$

$$=\frac{6h+h^2}{h}=\frac{\cancel{h}(6+h)}{\cancel{h}}$$

$= 6 + h$

$$f'(3) = \lim_{h \to 0} 6 + h$$

$= 6$

At $x = 1$?

$$\frac{f(1+h) - f(1)}{h}$$

$$= \frac{(1+h)^2 - 1^2}{h} = \frac{h(h+2)}{h}$$

$$= h + 2$$

$$f'(1) = \lim_{h \to 0} h + 2$$

$$= 2$$

For any x?

$$\frac{f(x+h)-f(x)}{h}$$

$$= \frac{(x+h)^2 - x^2}{h}$$

$$= \cdots = 2x+h$$

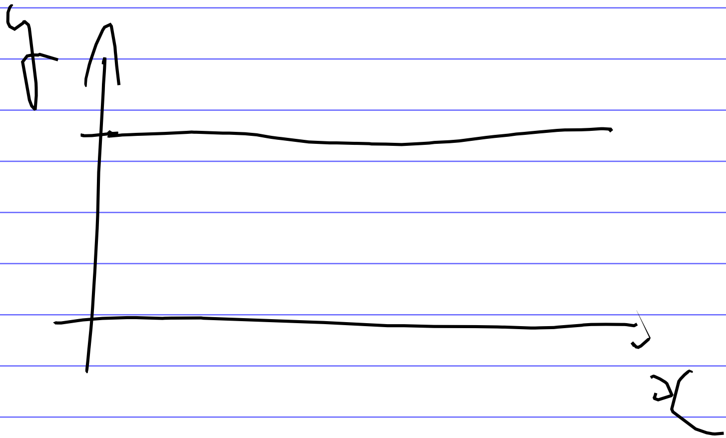
$$f'(x) = \lim_{h \to 0} 2x + h$$

$$= 2x$$

When $f(x) = x^2$

Then $f'(x) = 2x$

- $f(x) = 10$



- $f(x) = 2x^2$

$f'(x) = 2 \cdot 2x$

$= 4x$

$$f(x) = x^4$$

$$f'(x) = 4x^{4-1}$$

$$= 4x^3$$

$$f(x) = x^{100}$$

$$f'(x) = 100x^{99}$$

$$f(x) = 4x^2 + 5x + 10$$

$$f'(x) = 4 \cdot 2x^{2-1} + 5 + 0$$

$$= 8x + 5$$