SSE Riga - Maths Foundation

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February 8, 2020

Math Foundation

- 3 sessions :
  - February 8
  - February 15
  - February 22
- Starts at 10 :00
- Lecture + seminar
- Lecture slides + problem sets + solutions available online

Outline

- Session 1 : Introduction to differentiation
- Session 2 : Introduction to optimimization
- Session 3 : Introduction to integral calculus

Introduction to differentiation

Definition

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In economics :

- $Q_D(P)$: demand function
- $U(x)$: utility function
- $\Pi(Q)$: profit function
- ...

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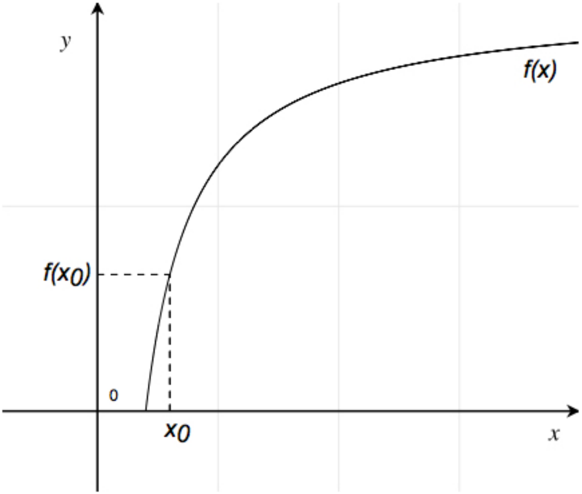
- Example : $Q_d(P) = -0,15P + 0,14$ represents the demand function for chocolate, with P in euro and Q in kg.

How to measure the rate of change when the function is not linear ?

What is the slope at $x = x_0$?



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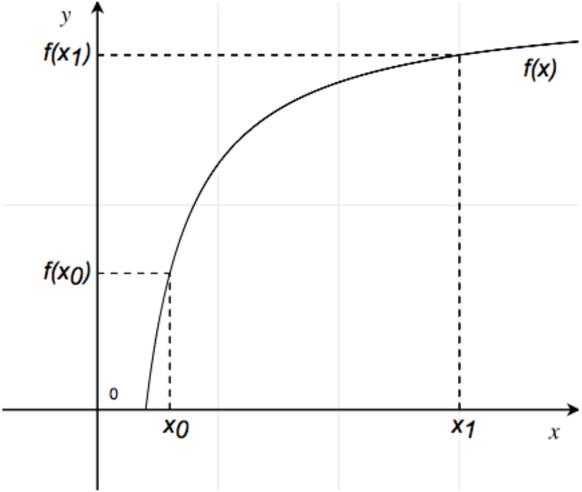

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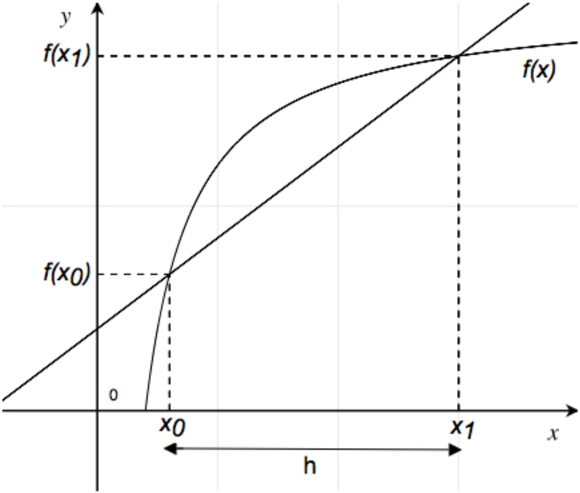

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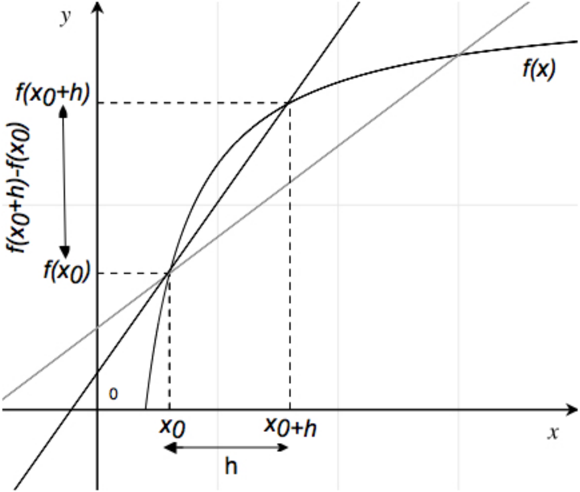

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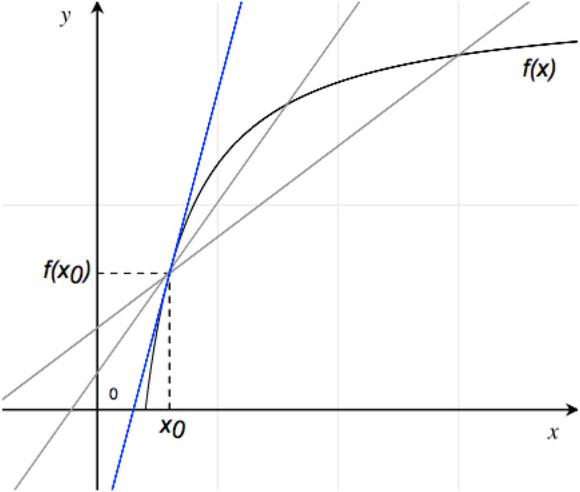

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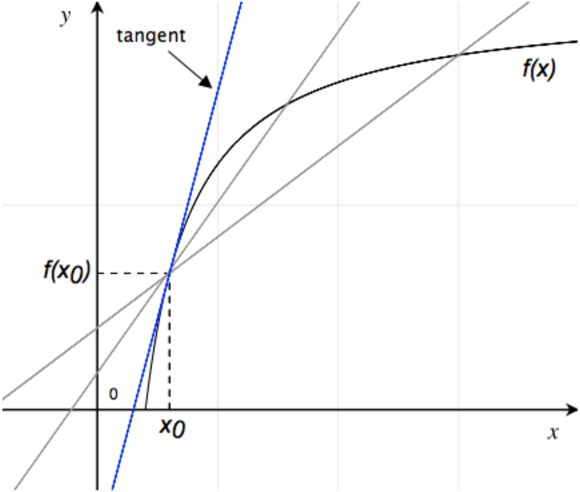

This is the most important slide of your life

definition

Let $(x_0, f(x_0))$ be a point on the graph of $y = f(x)$. The **derivative** of f at x_0 is the slope of the tangent line to the graph of f at $(x_0, f(x_0))$. We write :

$$f'(x_0) = \lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h}$$

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- Power rule :

$$\text{If } f(x) = x^a, \text{ then } f'(x) = ax^{a-1}$$

Differentiation of sums and differences

Consider the two differentiable functions $u(x)$ and $v(x)$

- If $f = u + v$, then $f' = u' + v'$
- If $f = u - v$, then $f' = u' - v'$

Application

The profit of your company is approximated by the following function :

$$f(x) = -x^3 + 40x^2,$$

where x stands for the quantity of units produced.
What is the effect of slightly increasing your production when you already produce 10 units ? When you produce 30 units ?

Application

$$f(x) = -x^3 + 40x^2 \Rightarrow f'(x) = -3x^2 + 80x,$$

- $f'(10) = -300 + 800 = 500$
- $f'(30) = -2700 + 2400 = -300$

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Thank you for your attention !